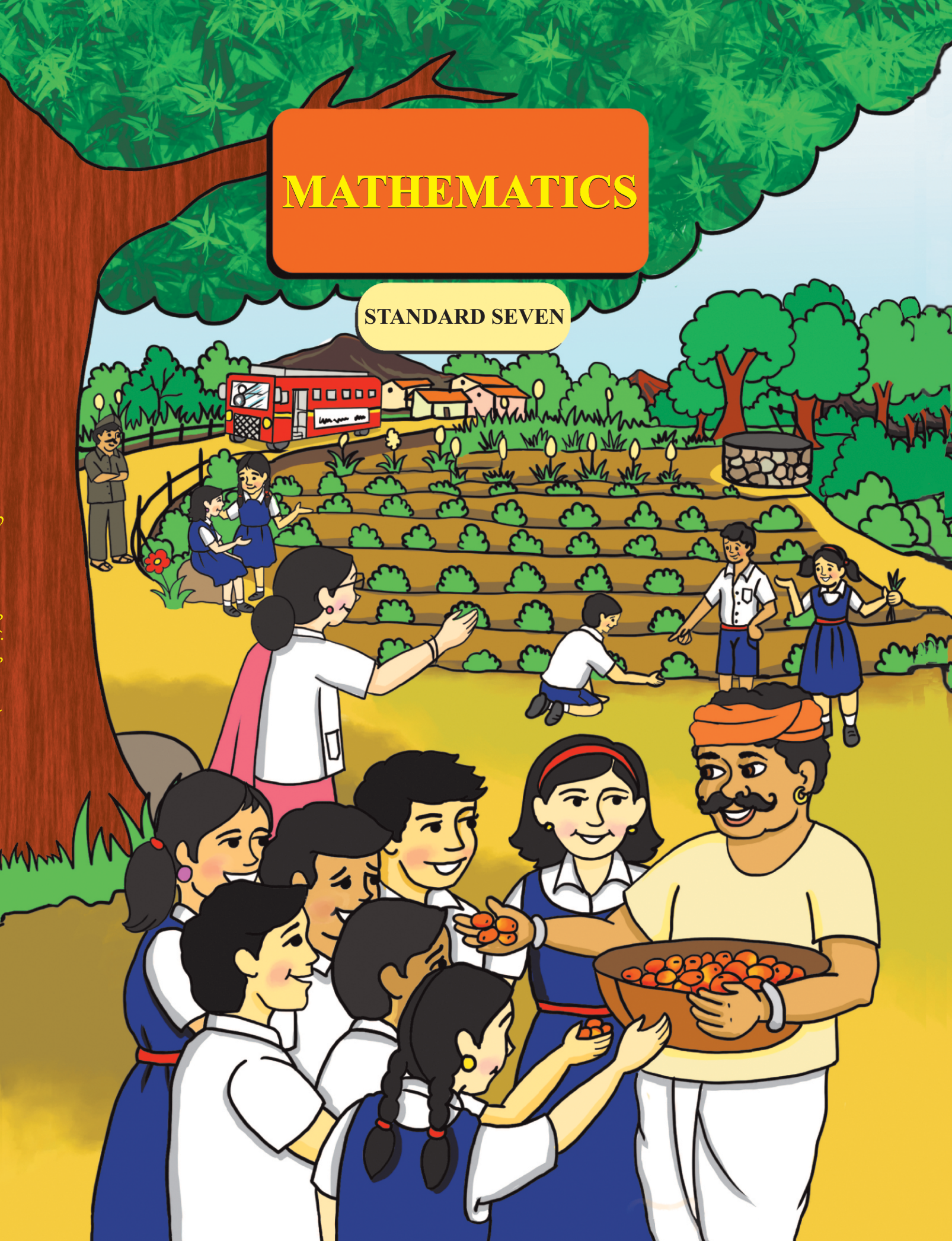
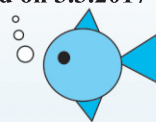
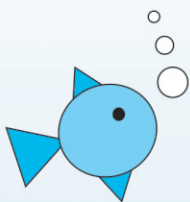


MATHEMATICS

STANDARD SEVEN

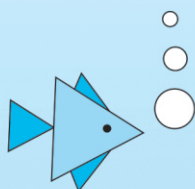
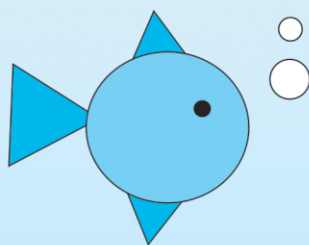
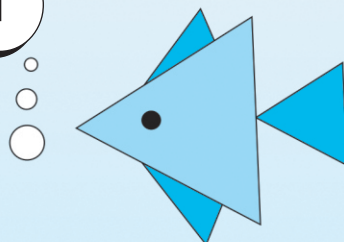


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Dated 25.4.2016 has given approval to prescribe this textbook in its meeting held on 3.3.2017



MATHEMATICS

STANDARD SEVEN



**Maharashtra State Bureau of Textbook Production and
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YH6VYA

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The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens :

JUSTICE, social, economic and political ;

LIBERTY of thought, expression, belief, faith and worship ;

EQUALITY of status and of opportunity ; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation ;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā
uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians
are my brothers and sisters.

I love my country, and I am proud
of its rich and varied heritage. I shall
always strive to be worthy of it.

I shall give my parents, teachers
and all elders respect, and treat
everyone with courtesy.

To my country and my people,
I pledge my devotion. In their
well-being and prosperity alone lies
my happiness.

PREFACE

Dear Students,

A warm welcome to all of you in Std VII! You have studied your maths textbooks of Stds I to VI. We are now happy to offer you the Std VII maths textbook.

We want you to understand maths well and also find it interesting. We want you to experience the joy of learning new things and finding answers to new questions. So do carry out all the activities and constructions given in the book for this very purpose. By doing so, you may just find out something interesting or some new mathematical properties! Discussing amongst yourselves helps to understand new concepts well. Pictures, Venn diagrams, the Internet, all help to understand mathematical concepts better. And if you understand the basic concepts, maths is not difficult at all. We expect that you will yourself read every chapter carefully. If you find something difficult, ask for help from your teachers or parents or other students in order to understand that part. The method of solving problems is given in the book along with the explanation of how and why a particular formula is obtained. Practise solving the problems by the given methods. This is very important. Design more problems of your own like the ones given in the Practice Sets. In this book, the more challenging problems have been marked with a star. The matter in the boxes will be of use to you for the maths studies that will follow. All the maths you have learnt since Std I, you will find useful in the future too. For example, you can hardly afford to forget addition, subtraction, multiplication and division! Practice these operations till you are really good at them. You have to use them all many times while solving problems later.

Many basic concepts have been introduced in this Std VII book. If you gain a good understanding of them all, you will find maths easy in the following years.

Come then, this book is looking forward to be your companion and friend as you make your efforts to understand and learn mathematics.

Pune

Date : 28 March 2017

Indian Solar Year:

7 Chaitra 1939



(Dr Sunil Magar)

Director

Maharashtra State Bureau of Textbook
Production and Curriculum Research, Pune.

English Mathematics - Standard VII

Learning Outcomes

Suggested Pedagogical Processes	Learning Outcomes
<p>The learner may be provided opportunities in pairs/groups/ individually and encouraged to —</p> <ul style="list-style-type: none"> provide contexts for exploring the rules of multiplication and division of integers. This can be done through number-line or number patterns. For example : $3 \times 2 = 6$ $3 \times 1 = 3$ $3 \times 0 = 0$ $3 \times (-1) = -3$ $3 \times (-2) = -6$ <p>Same number reduced by one number number reduced by 3</p> <p>So $3 \times (-3) = -9$ Means a positive integer multiplied by a negative integer given a negative integer For example: (a) $\frac{1}{4} \times \frac{1}{2}$ is $\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{8}$ (b) $\frac{1}{2} \div \frac{1}{4}$ means number of $\frac{1}{4}$ in $\frac{1}{2}$ are two</p>  <ul style="list-style-type: none"> explore the multiplication/ division of fractions/ decimals through pictures/paper folding activities of daily life examples. discuss the situations that require the use of fractional numbers in opposite direction, such as moving $10\frac{1}{2}$ m to the right of a tree and $15\frac{2}{3}$ m to its left etc. involve children in exploring how repeated multiplication of numbers can be expressed in short form. For example, $2 \times 2 \times 2 \times 2 \times 2 \times 2 =$ can be expressed as 2^6. explore the possible combinations of variables and constants using different operations to form algebraic expressions in various contexts. provide situations from daily life that lead to setting up of equations and choosing the appropriate value of the variable that equate both sides. conduct activity of adding /subtracting number of objects of same category from daily life. For example, number of notebooks obtained when 3 notebooks are added to a group of 5 notebooks. evolve the understanding of the concepts of ratios and percentage (equality of ratio). 	<p>The learner —</p> <p>07.71.01 multiplies/divides two integers.</p> <p>07.71.02 interprets the division and multiplication of fractions. For example, interprets $\frac{2}{3} \times \frac{4}{5}$ as $\frac{2}{3}$ of $\frac{4}{5}$. Also $\frac{1}{2} \div \frac{1}{4}$ is interpreted as how many $\frac{1}{4}$ make $\frac{1}{2}$?</p> <p>07.71.03 uses algorithms to multiply and divide fractions/ decimals.</p> <p>07.71.04 solves problems related to daily life situations involving rational numbers.</p> <p>07.71.05 uses exponential form of numbers to simplify problems involving multiplication and division of large numbers.</p> <p>07.71.06 represents daily life situations in the form of a simple equation and solves it.</p> <p>07.71.07 adds/subtracts algebraic expressions.</p> <p>07.71.08 distinguishes quantities that are in proportion. For example, tells that 15, 45, 40, 120 are in proportion as $\frac{15}{45}$ is the same as $\frac{40}{120}$.</p> <p>07.71.09 solves problems related to conversion of percentage to fraction and decimal and vice versa.</p> <p>07.71.10 classifies pairs of angles based on their properties as linear, supplementary, complementary, adjacent and vertically opposite and finds value of the one when the other is given.</p> <p>07.71.11 finds unknown angle of a triangle when its two angles are known.</p> <p>07.71.12 calculates areas of the regions enclosed in a rectangle and a square.</p> <p>07.71.13 finds various representative values for simple data from her/his daily life contexts like mean, median and mode.</p> <p>07.71.14 interprets data using bar graph such as consumption of electricity is more in winters than summer, runs scored by a team in first 10 overs etc.</p> <p>07.71.15 constructs angle bisector and perpendicular bisector at sides of a triangle and recognises that they are concurrent.</p> <p>07.71.16 constructs triangle if particular sides and angles are given.</p> <p>07.71.17 identifies congruence of angles, segments and circles.</p>

<ul style="list-style-type: none"> • provide daily life situations based on profit/loss and simple interest that show the use of percentage • explore different examples from daily life in which pair of angles are involved with a common vertex, for example, Scissors, Road Junction, Letter X, T, etc • verify the properties of various pairs of angles by drawing diagram (One group can give measure of one angle, the other group needs to give the measure of other angle.) • visualise the relationship between various pairs of angles when a transversal cuts two lines (parallel and non-parallel), angles of triangle and relationship among its sides through diagrams and upper primary Mathematics kit • draw different types of triangles, ask them to measure angles of all triangles and verify. • explore exterior angle property of triangles and Pythagoras theorem. • establish congruence criterion and later on verify the property by superimposing one above the other • construct the simple triangle by using ruler and compasses. • find a representative value of data i.e. mean, mode or median of ungrouped data. Encourage them to arrange it in a tabular form and represent it by bar graphs • draw inferences for future events from the existing data. • understand the property that sum of two sides of a triangle is greater than the third side. 	<ul style="list-style-type: none"> 07.71.18 calculate GCD and LCM by using prime factorization. 07.71.19 identifies exterior angles of triangle. 07.71.20 derives the formula for sum of interior angles of polygon. 07.71.21 calculate the square root of number by prime factorization. 07.71.22 draws and reads joint bar graphs from the given information. 07.71.23 uses the properties of proportion in partnership. 07.71.24 derives the formula for circumference and applies. 07.71.25 identifies the minor and major arc and finds measure of them. 07.71.26 derives the formula for area of triangle. 07.71.27 finds surface area of cube and cuboid. 07.71.28 uses the Pythagoras theorem for finding one side of right angled triangle. 07.71.29 uses formula for expansion of square. 07.71.30 writes the squares of any binomial. 07.71.31 finds the factors of binominal.
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Guidelines for Teachers

The Std VII textbook needs to be used for question-answers, activities and conversations with students in the class. Hence, it should be read very thoroughly. The textbook relates mathematics with all other subjects such as our environment, geography, science, economics. Teachers should show the students how mathematics is used in all these various subjects. This will impress upon them the usefulness of mathematics in daily affairs and will convince them of the importance of studying the subject. Explanations of the mathematical concepts have been given in simple language. Teachers should frame more problems based on those included in the practice sets and encourage students also to do the same.

Some challenging problems have been provided for the students. Those problems are marked with a star. Some extra information is given under the title 'Something more'. Students may find it useful for their further studies. We hope and believe that you will like the new Std VII maths textbook.

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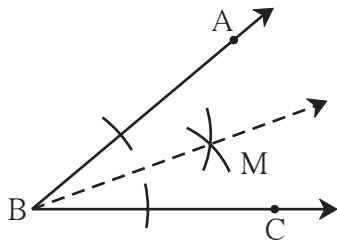
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Let's recall.

In previous classes, we have learnt about the line, line segment, angle, angle bisector, etc. We measure an angle in degrees. If $\angle ABC$ measures 40° , we write it as $m\angle ABC = 40^\circ$.



Angle Bisector

You see the figure of $\angle ABC$ alongside.

An angle bisector divides an angle into two equal parts. Is ray BM the bisector of $\angle ABC$?

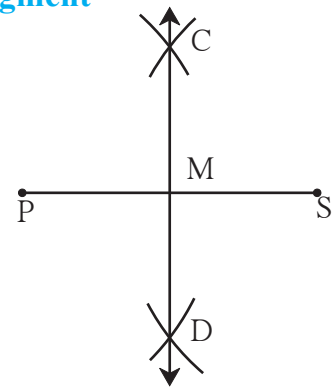
Perpendicular Bisector of a Line Segment

Draw a line segment PS of length 4 cm and draw its perpendicular bisector. Name it line CD.

- How will you verify that CD is the perpendicular bisector?

$$m\angle CMS = \boxed{}^\circ$$

- Is $l(PM) = l(SM)$?

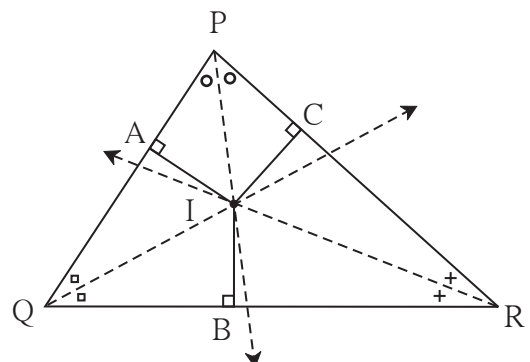


Let's learn.

The Property of the Angle Bisectors of a Triangle

Activity

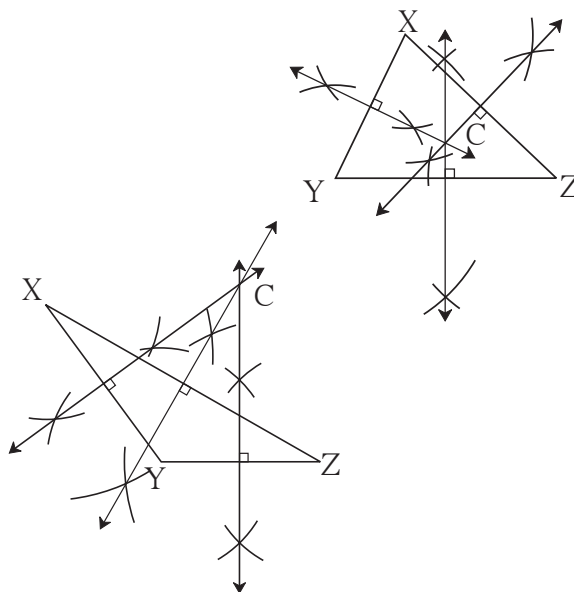
- Draw any $\triangle PQR$.
- Use a compass to draw the bisectors of all three of its angles. (Extend the bisectors, if necessary, so that they intersect one another.)
- These three bisectors pass through the same point. That is, they are **concurrent**. Name the point of concurrence 'I'. Note that the point of concurrence of the angle bisectors of a triangle is in the interior of the triangle.
- Draw perpendiculars IA, IB and IC respectively from I on to the sides of the triangle PQ, QR and PR. Measure the lengths of these perpendiculars. Note that $IA = IB = IC$.



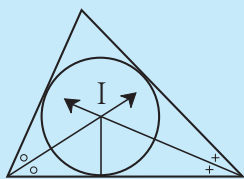
The Property of Perpendicular Bisectors of the Sides of a Triangle

Activity

1. Use a ruler to draw an acute-angled triangle and an obtuse-angled triangle. Draw the perpendicular bisectors of each side of the two triangles.
2. In each triangle, note that the perpendicular bisectors of the sides are concurrent.
3. Name their point of concurrence 'C'. Measure the distance between C and the vertices of the triangle. Note that $CX = CY = CZ$.
4. Observe the location of the point of concurrence of the perpendicular bisectors.

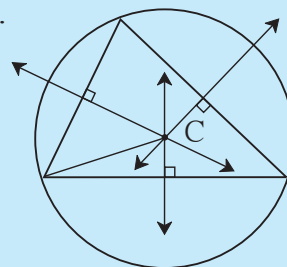


★ Something more



- (1) The angle bisectors of a triangle are **concurrent**. Their point of concurrence is called the **incentre**, and is shown by the letter 'I'.

- (2) The perpendicular bisectors of the sides of a triangle are **concurrent**. Their point of concurrence is called the **circumcentre** and is shown by the letter 'C'.



Practice Set 1

1. Draw line segments of the lengths given below and draw their perpendicular bisectors.
(1) 5.3 cm (2) 6.7 cm (3) 3.8 cm
2. Draw angles of the measures given below and draw their bisectors.
(1) 105° (2) 55° (3) 90°
3. Draw an obtuse-angled triangle and a right-angled triangle. Find the points of concurrence of the angle bisectors of each triangle. Where do the points of concurrence lie?
4. Draw a right-angled triangle. Draw the perpendicular bisectors of its sides. Where does the point of concurrence lie?
- 5*. Maithili, Shaila and Ajay live in three different places in the city. A toy shop is equidistant from the three houses. Which geometrical construction should be used to represent this? Explain your answer.



Let's learn.

Construction of a Triangle

Activity

Let us see if we can draw the triangles when the measures of some sides and angles, are given.

Draw $\triangle ABC$ such that

$l(AB) = 4$ cm, and $l(BC) = 3$ cm.

- Can this triangle be drawn?
- A number of triangles can be drawn to fulfil these conditions. Try it out.
- Which further condition must be placed if we are to draw a unique triangle using the above information?

Before constructing any building, the structure of the building is first drawn on paper. You might have even seen a small model of some such building. It becomes easier to construct the building from a drawing. In a similar way, a rough sketch of a geometrical construction makes it easier to draw the required figure. It helps to plan the sequence of the steps in the construction.

(I) To construct a triangle given the lengths of its three sides

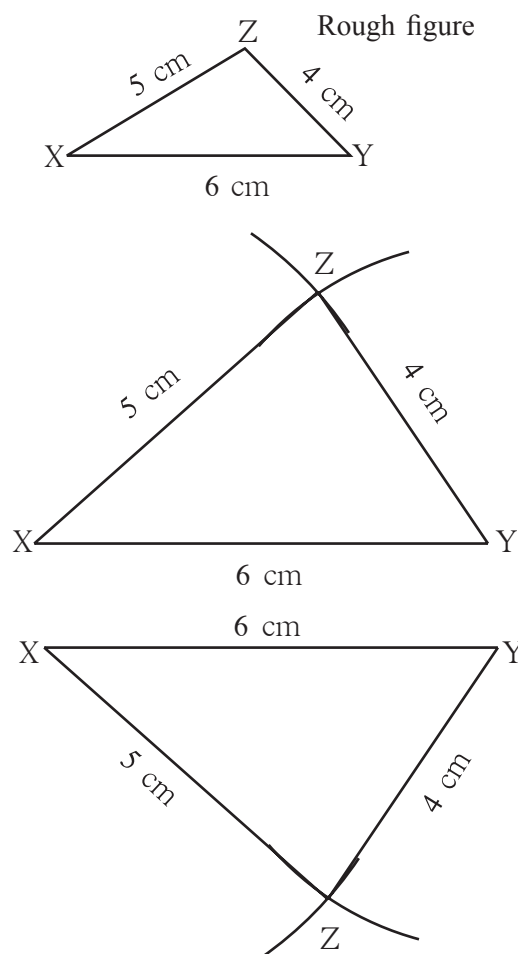
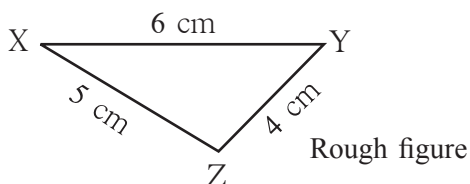
Example Draw $\triangle XYZ$ such that $l(XY) = 6$ cm, $l(YZ) = 4$ cm, $l(XZ) = 5$ cm

- Let us draw a rough figure quickly and show the given information in it as accurately as possible. For example, side XY is the longest, so, in the rough figure, too, XY should be the longest side.

Steps

1. According to the rough figure, segment XY of length 6 cm is drawn as the base.
2. As $l(XZ)$ is 5 cm, draw an arc on one side of seg XY with the compass opened to 5 cm and with its point at X .
3. Next, with the point at Y and the compass opened to 4 cm, draw an arc to cut the first arc at Z . Draw segs XY and YZ .

A similar construction can be drawn on the other side of the base as shown below.



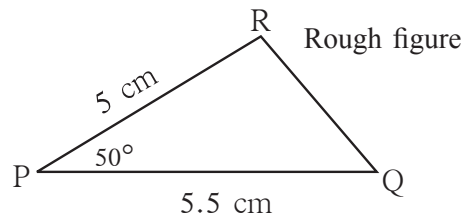
Practice Set 2

- Draw triangles with the measures given below.
 - In $\triangle ABC$, $l(AB) = 5.5$ cm, $l(BC) = 4.2$ cm, $l(AC) = 3.5$ cm
 - In $\triangle STU$, $l(ST) = 7$ cm, $l(TU) = 4$ cm, $l(SU) = 5$ cm
 - In $\triangle PQR$, $l(PQ) = 6$ cm, $l(QR) = 3.8$ cm, $l(PR) = 4.5$ cm
- Draw an isosceles triangle with base 5 cm and the other sides 3.5 cm each.
- Draw an equilateral triangle with side 6.5 cm.
- Choose the lengths of the sides yourself and draw one equilateral, one isosceles and one scalene triangle.

(II) To construct a triangle given two sides and the angle included by them

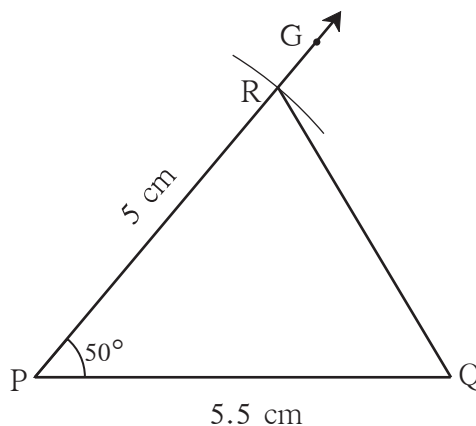
Example Draw $\triangle PQR$ such that $l(PQ) = 5.5$ cm, $m\angle P = 50^\circ$, $l(PR) = 5$ cm.

(A rough figure has been drawn showing the given information. $\angle P$ is an acute angle and that is shown in the rough figure, too.)

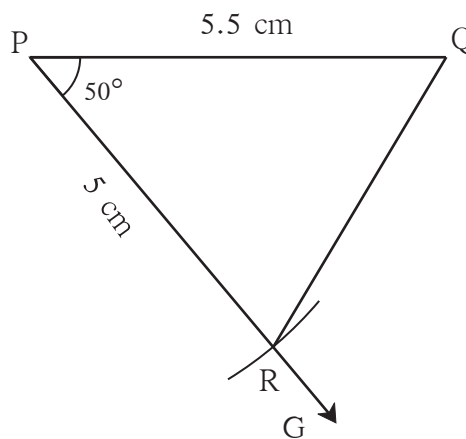
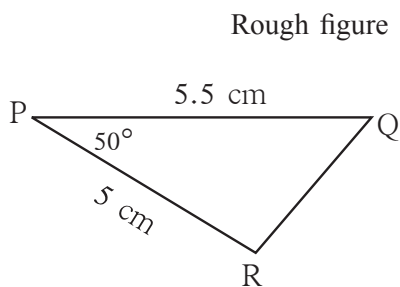


Steps

- According to the rough figure, seg PQ forms the base of length 5.5 cm.
- Ray PG is drawn so that $m\angle GPQ = 50^\circ$
- Open the compass to 5 cm. Placing the compass point on P, draw an arc to cut ray PG at R. Join points Q and R. $\triangle PQR$ is the required triangle.



The ray PG may be drawn on the other side of the seg PQ. Its rough figure will be as shown below.



Practice Set 3

☉ Draw triangles with the measures given below.

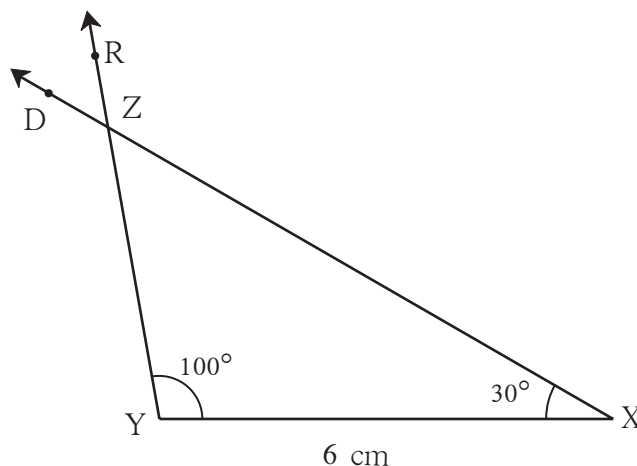
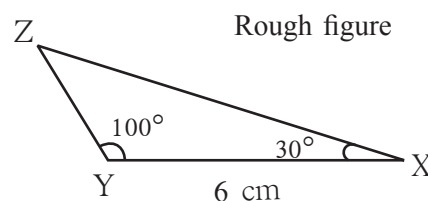
1. In $\triangle MAT$, $l(MA) = 5.2$ cm,
 $m\angle A = 80^\circ$, $l(AT) = 6$ cm
2. In $\triangle NTS$, $m\angle T = 40^\circ$,
 $l(NT) = l(TS) = 5$ cm
3. In $\triangle FUN$, $l(FU) = 5$ cm,
 $l(UN) = 4.6$ cm, $m\angle U = 110^\circ$
4. In $\triangle PRS$, $l(RS) = 5.5$ cm,
 $l(RP) = 4.2$ cm, $m\angle R = 90^\circ$

(III) To construct a triangle given two angles and the included side

Example Construct $\triangle XYZ$ such that $l(YX) = 6$ cm, $m\angle ZXY = 30^\circ$, $m\angle XYZ = 100^\circ$
 $\angle XYZ$ is an obtuse angle and that is shown in the rough figure.

Steps

1. According to the rough figure, draw seg YX as base of length 6 cm.
 2. Draw ray YR such that
 $m\angle XYR = 100^\circ$
 3. On the same side of seg YX as point R, draw ray XD so that
 $m\angle YXD = 30^\circ$. Name the point of intersection of rays YR and XD, Z.
- $\triangle XYZ$ is the required triangle.
4. See how an identical triangle can be drawn on the other side of the base.



Use your brain power.

Example In $\triangle ABC$, $m\angle A = 60^\circ$, $m\angle B = 40^\circ$, $l(AC) = 6$. Can you draw $\triangle ABC$?

What further information is required before it can be drawn? Which property can be used to get it? Draw the rough figure to find out.

Recall the property of the sum of the angles of a triangle. Using that property, can we find the measures of two angles and an included side AC ?

Practice Set 4

☉ Construct triangles of the measures given below.

1. In $\triangle SAT$, $l(AT) = 6.4$ cm,
 $m\angle A = 45^\circ$, $m\angle T = 105^\circ$
2. In $\triangle MNP$, $l(NP) = 5.2$ cm,
 $m\angle N = 70^\circ$, $m\angle P = 40^\circ$
3. In $\triangle EFG$, $l(EG) = 6$ cm,
 $m\angle F = 65^\circ$, $m\angle G = 45^\circ$
4. In $\triangle XYZ$, $l(XY) = 7.3$ cm,
 $m\angle X = 34^\circ$, $m\angle Y = 95^\circ$

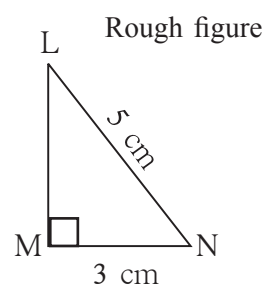
(IV) To construct a right-angled triangle given the hypotenuse and one side

We know that a triangle with a right angle is called a right-angled triangle. In such a triangle, the **side opposite the right angle** is called the **hypotenuse**.

Example Draw $\triangle LMN$ such that $m\angle LMN = 90^\circ$, hypotenuse = 5 cm, $l(MN) = 3$ cm.

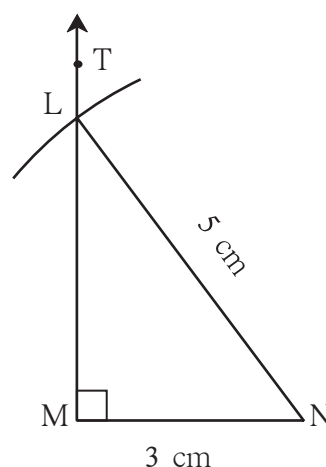
Let us draw the rough figure using the given information.

As $m\angle LMN = 90^\circ$, we draw a right-angled triangle approximately and mark the right angle. Thus we show the given information in the rough figure.



Steps

1. As shown in the rough figure, draw the base seg MN of length 3 cm.
2. At point M of seg MN, draw ray MT to make an angle of 90° to seg MN.
3. Opening the compass to 5 cm and with the point at N, draw an arc to cut seg MT at L. $\triangle LMN$ is the required triangle.
4. Note that a similar figure can be drawn on the other side of the base.



Practice Set 5

☉ Construct triangles of the measures given below.

1. In $\triangle MAN$, $m\angle MAN = 90^\circ$,
 $l(AN) = 8$ cm, $l(MN) = 10$ cm.
2. In the right-angled $\triangle STU$, hypotenuse
 $SU = 5$ cm and $l(ST) = 4$ cm.
3. In $\triangle ABC$, $l(AC) = 7.5$ cm,
 $m\angle ABC = 90^\circ$, $l(BC) = 5.5$ cm.
4. In $\triangle PQR$, $l(PQ) = 4.5$ cm,
 $l(PR) = 11.7$ cm, $m\angle PQR = 90^\circ$.
5. Students should take examples of their own and practise construction of triangles.

Activity

Try to draw triangles with the following data.

1. $\triangle ABC$ in which $m\angle A = 85^\circ$, $m\angle B = 115^\circ$, $l(AB) = 5$ cm
2. $\triangle PQR$ in which $l(QR) = 2$ cm, $l(PQ) = 4$ cm, $l(PR) = 2$ cm

Could you draw these triangles? If not, look for the reasons why you could not do so.

* An activity for learning something more

Example Draw $\triangle ABC$ such that $l(BC) = 8$ cm, $l(CA) = 6$ cm, $m\angle ABC = 40^\circ$.

Draw a ray to make an angle of 40° with the base BC, $l(BC) = 8$ cm.

We have to obtain point 'A' on the ray. With 'C' as the centre, draw an arc of radius 6 cm to do so. What do we observe? The arc intersects the ray in two different points. Thus, we get two triangles of two different shapes having the given measures.

Can a triangle be drawn if the three angles are given, but not any side?
How many such triangles can be drawn?

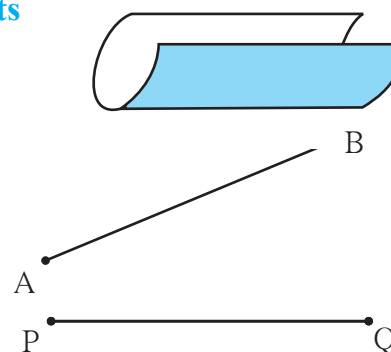


Let's learn.

Congruence of Segments

Activity I Take a rectangular paper. Place two opposite sides one upon the other. They coincide exactly.

Activity II Using the ruler measure the lengths of seg AB and seg PQ.
 $l(AB) = \dots\dots\dots$ $l(PQ) = \dots\dots\dots$



Are they of the same length? You cannot pick up and place one segment over the other. Trace the seg AB along with the names of the points on a sheet of transparent paper. Place this new segment on seg PQ. Verify that if point A is placed on point P, then B falls on Q. It means that seg AB is **congruent** with seg PQ.

We can infer from this that if two line segments have the same lengths, they coincide exactly with each other. That is, they are **congruent**. If seg AB and seg PQ are congruent, it is written as $\text{seg AB} \cong \text{seg PQ}$.



Now I know!

If given line segments are equal in length, they are congruent.

☼ If $\text{seg AB} \cong \text{seg PQ}$ it means that $\text{seg PQ} \cong \text{seg AB}$.

☼ Note also that if $\text{seg AB} \cong \text{seg PQ}$ and $\text{seg PQ} \cong \text{seg MN}$, then $\text{seg AB} \cong \text{seg MN}$.

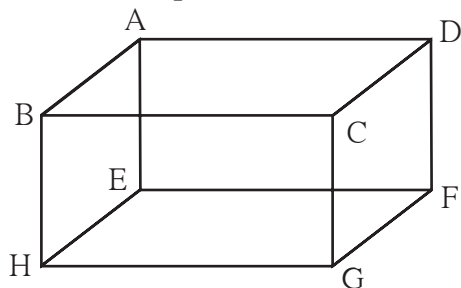
In other words, if one line segment is congruent to another and that segment is congruent to a third, then the first segment is also congruent to the third.

Activity I

Take any box. Measure the lengths of each of its edges. Which of them are congruent?

Activity II

From the shape shown below, write the names of the pairs of congruent line segments.



(1) $\text{seg } AB \cong \text{seg } DC$

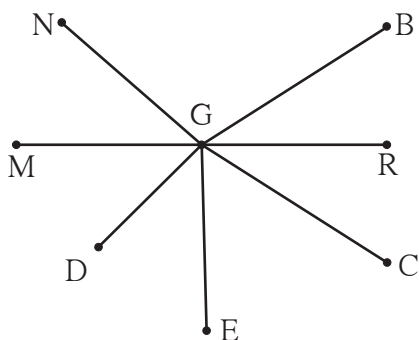
(2) $\text{seg } AE \cong \text{seg } BH$

(3) $\text{seg } EF \cong \text{seg } \dots\dots$

(4) $\text{seg } DF \cong \text{seg } \dots\dots$

Practice Set 6

1. Write the names of pairs of congruent line segments. (Use a divider to find them.)



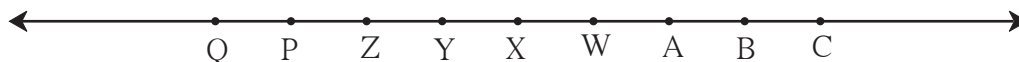
(i) $\dots\dots\dots$

(ii) $\dots\dots\dots$

(iii) $\dots\dots\dots$

(iv) $\dots\dots\dots$

2. On the line below, the distance between any two adjoining points shown on it is equal. Hence, fill in the blanks.



(i) $\text{seg } AB \cong \text{seg } \dots\dots\dots$

(ii) $\text{seg } AP \cong \text{seg } \dots\dots\dots$

(iii) $\text{seg } AC \cong \text{seg } \dots\dots\dots$

(iv) $\text{seg } \dots\dots\dots \cong \text{seg } BY$

(v) $\text{seg } \dots\dots\dots \cong \text{seg } YQ$

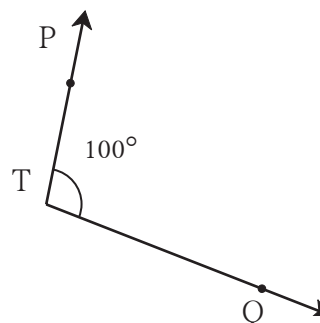
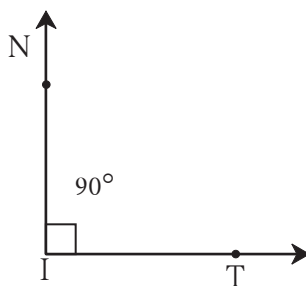
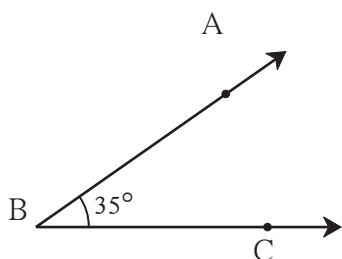
(vi) $\text{seg } BW \cong \text{seg } \dots\dots\dots$

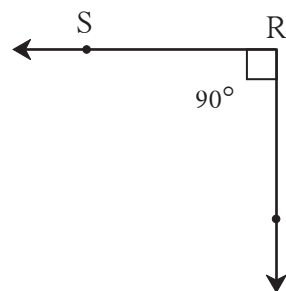
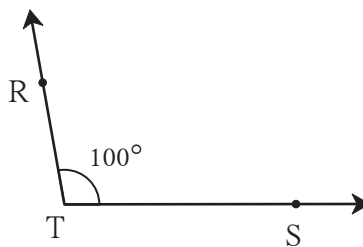
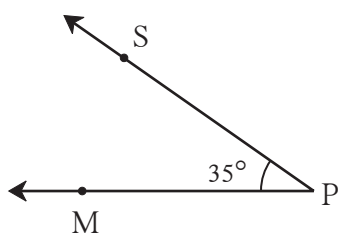


Let's learn.

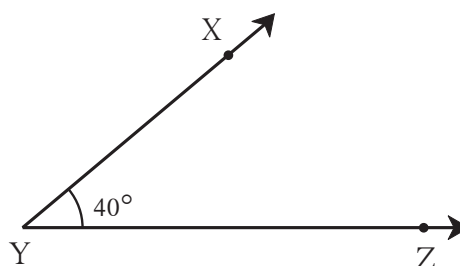
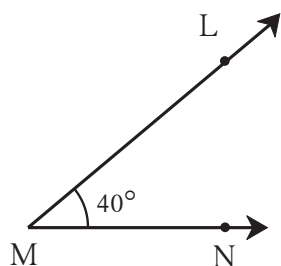
Congruence of Angles

Observe the given angles and write the names of those having equal measures.





Activity



Draw two angles $\angle LMN$ and $\angle XYZ$ of 40° each as shown in the figure. Trace the arms of $\angle LMN$ and the names of the points on a transparent paper. Now lift the transparent paper and place the angle you obtain on $\angle XYZ$. Observe that if point M is placed on Y and ray MN on ray YZ, then ray ML falls on ray YX. We can infer that angles of equal measure are congruent. The congruence of angles does not depend on the length of their arms. It depends upon the measures of those angles. That $\angle LMN$ is congruent with $\angle XYZ$ is written as $\angle LMN \cong \angle XYZ$.



Now I know!

Two angles with equal measures are congruent to each other.



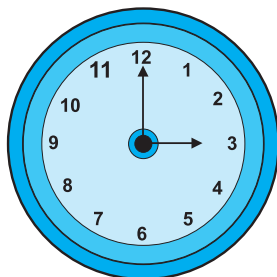
If $\angle LMN \cong \angle XYZ$ then $\angle XYZ \cong \angle LMN$.



If $\angle LMN \cong \angle ABC$ and $\angle ABC \cong \angle XYZ$ then $\angle LMN \cong \angle XYZ$.



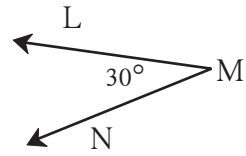
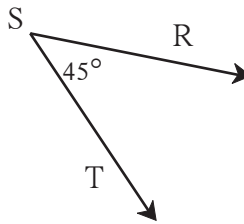
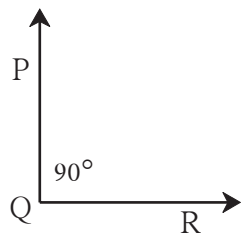
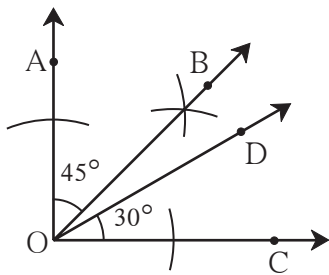
Let's discuss.



- (1) What time does this clock show?
- (2) What is the measure of the angle between its two hands?
- (3) At which other times is the angle between the hands congruent with this angle?

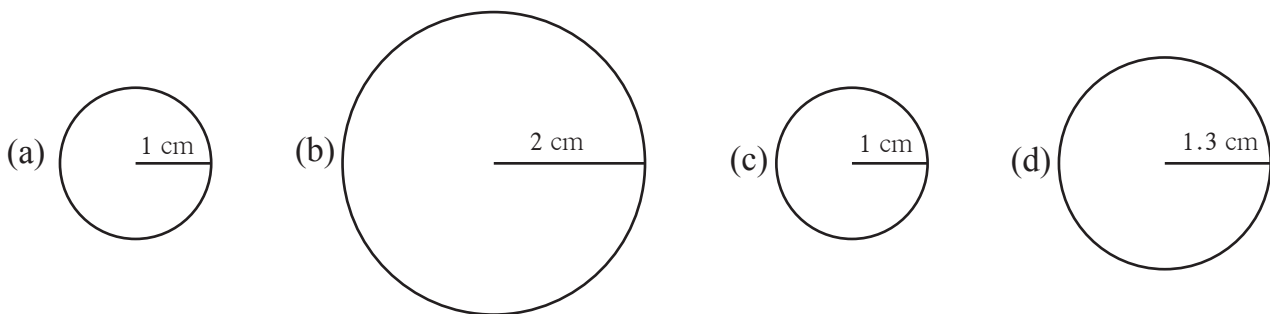
Practice Set 7

- Some angles are given below. Using the symbol of congruence write the names of the pairs of congruent angles in these figures.



Let's learn.

Congruence of Circles



Activity I

Observe the circles in the figures above. Draw similar circles of radii 1 cm, 2 cm, 1 cm and 1.3 cm on a paper and cut out these circular discs. Place them one upon the other to find out which ones coincide exactly.

Observations: 1. The circles in (a) and (c) coincide.

2. Circles in fig (b) and (c) and in fig (a) and (d) do not coincide.

Are there other pairs like these ?

Circles which coincide exactly are said to be **congruent circles**.

Activity II Get bangles of different sizes but equal thickness and find the congruent ones among them.

Activity III Find congruent circles in your surroundings.

Activity IV Take some round bowls and plates. Place their edges one upon the other to find pairs of congruent edges.



Now I know!

Circles of equal radii are congruent circles.



ICT Tools or Links

Use the construction tools in the Geogebra software to draw triangles and circles.



**Let's recall.**

- In the previous class, we have learnt to add and subtract integers. Using those methods, fill in the blanks below.

(1) $5 + 7 = \square$

(2) $10 + (-5) = \square$

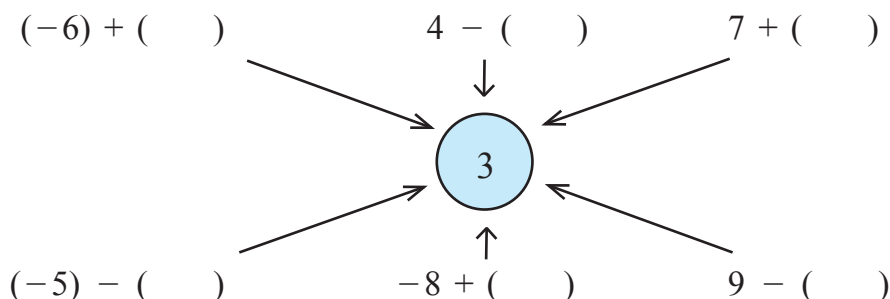
(3) $-4 + 3 = \square$

(4) $(-7) + (-2) = \square$

(5) $(+8) - (+3) = \square$

(6) $(+8) - (-3) = \square$

- Write a number in each bracket to obtain the answer '3' in each operation.

**Let's learn.****Multiplication of Integers**

Mayuri's bicycle got punctured on the way back from school and she did not have enough money to get it repaired. Sushant, Snehal and Kalpana lent her five rupees each. Thus she borrowed 15 rupees altogether and got the bicycle repaired. We show borrowed money, or a debt, using the '-' (minus) sign. That is, Mayuri had a debt of 15 rupees or Mayuri had -15 rupees.

We see here that $(-5) + (-5) + (-5) = -15$

Hence note that $(-5) \times 3 = 3 \times (-5) = -15$

Of course, Mayuri paid back her debt the next day.

We have learnt the multiplication and division of whole numbers. We have even made tables to carry out the multiplication. Now let us learn to multiply integers i.e. multiplication of numbers in the set that includes negative numbers, positive numbers and zero.

$(-3) + (-3) + (-3) + (-3)$ This addition is the addition of (-3) taken 4 times. It equals -12 . It can be written as $(-3) \times 4 = -12$.

Similarly, $(-5) \times 6 = -30$, $(-7) \times 2 = -14$, $8 \times (-7) = -56$

Now, let us make the table of (-4) .

$$\begin{aligned} (-4) \times 0 &= 0 \\ (-4) \times 1 &= -4 \\ (-4) \times 2 &= -8 \\ (-4) \times 3 &= -12 \end{aligned}$$



Observe the pattern here. As the multiplier of (-4) increases by 1, the product is reduced by 4.

Keeping the same pattern, if we extend the table upwards, decreasing the multiplier, this is what we will get.

$$\begin{aligned} (-4) \times (-2) &= 8 \\ (-4) \times (-1) &= 4 \\ (-4) \times 0 &= 0 \end{aligned}$$



As the multiplier of (-4) decreases by one unit, the product increases by 4.

The table for (-5) is given below. Complete the tables of (-6) and (-7) .

$(-5) \times (-3) = 15$	$(-6) \times (-3) =$ <input type="text"/>	$(-7) \times (-3) =$ <input type="text"/>
$(-5) \times (-2) = 10$	$(-6) \times (-2) =$ <input type="text"/>	$(-7) \times (-2) =$ <input type="text"/>
$(-5) \times (-1) = 5$	$(-6) \times (-1) =$ <input type="text"/>	$(-7) \times (-1) =$ <input type="text"/>
$(-5) \times 0 = 0$	$(-6) \times 0 =$ <input type="text"/>	$(-7) \times 0 =$ <input type="text"/>
$(-5) \times 1 = -5$	$(-6) \times 1 =$ <input type="text"/>	$(-7) \times 1 =$ <input type="text"/>
$(-5) \times 2 = -10$	$(-6) \times 2 =$ <input type="text"/>	$(-7) \times 2 =$ <input type="text"/>
$(-5) \times 3 = -15$	$(-6) \times 3 =$ <input type="text"/>	$(-7) \times 3 =$ <input type="text"/>
$(-5) \times 4 = -20$	$(-6) \times 4 =$ <input type="text"/>	$(-7) \times 4 =$ <input type="text"/>



Now I know!

- The product of two positive (+ve) integers is a positive (+ve) integer.
- The product of one positive (+ve) and one negative (-ve) integer is a negative integer.
- The product of two negative (-ve) integers is a positive (+ve) integer.

$$(+ve \text{ number}) \times (+ve \text{ number}) = (+ve \text{ number})$$

$$(+ve \text{ number}) \times (-ve \text{ number}) = (-ve \text{ number})$$

$$(-ve \text{ number}) \times (+ve \text{ number}) = (-ve \text{ number})$$

$$(-ve \text{ number}) \times (-ve \text{ number}) = (+ve \text{ number})$$

Practice Set 8

⊙ Multiply.

(i) $(-5) \times (-7)$

(ii) $(-9) \times (6)$

(iii) $(9) \times (-4)$

(iv) $(8) \times (-7)$

(v) $(-124) \times (-1)$

(vi) $(-12) \times (-7)$

(vii) $(-63) \times (-7)$

(viii) $(-7) \times (15)$



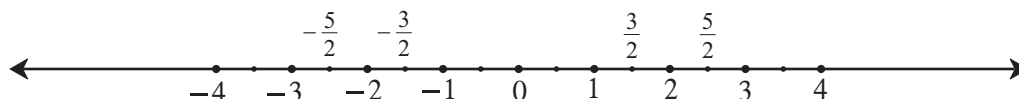
Let's learn.

Division of Integers

We have learnt how to divide one positive integer by another. We also know that the quotient of such a division may be an integer or a fraction.

Example $6 \div 2 = \frac{6}{2} = 3$, $5 \div 3 = \frac{5}{3} = 1 + \frac{2}{3}$

On the number line, we can show negative integers on the left of the zero. We can show parts of integers also in the same way.



Here, the numbers $-\frac{5}{2}$, $-\frac{3}{2}$, $\frac{3}{2}$, $\frac{5}{2}$ are shown on the number line.

Note that $\left(-\frac{1}{2}, \frac{1}{2}\right)$, $\left(\frac{3}{2}, -\frac{3}{2}\right)$, $\left(-\frac{5}{2}, \frac{5}{2}\right)$ are mutually opposite numbers.

That is, $\frac{1}{2} + \frac{-1}{2} = 0$, $\frac{3}{2} + \frac{(-3)}{2} = 0$, $-\frac{5}{2} + \frac{5}{2} = 0$

Pairs of opposite numbers are also called pairs of additive inverse numbers. We have seen that $(-1) \times (-1) = 1$. If the two sides of this equation are divided by (-1) we get the equation $(-1) = \frac{1}{(-1)}$. Therefore, the quotient of the division $\frac{1}{(-1)}$ is (-1) .

Hence, we see that $6 \times (-1) = 6 \times \frac{1}{(-1)} = \frac{6}{(-1)}$.

To divide any positive integer by a negative integer

$$\frac{7}{-2} = \frac{7 \times 1}{(-1) \times 2} = 7 \times \frac{1}{(-1)} \times \frac{1}{2} = \frac{7}{1} \times (-1) \times \frac{1}{2} = \frac{(7) \times (-1)}{2} = \frac{-7}{2}$$

To divide any negative integer by a negative integer

$$\frac{-13}{-2} = \frac{(-1) \times 13}{(-1) \times 2} = \frac{(-1)}{(-1)} \times 13 \times \frac{1}{2} = (-1) \times \frac{(-1)}{1} \times \frac{13}{2} = 1 \times \frac{13}{2} = \frac{13}{2}$$

Similarly, verify that $\frac{-25}{-4} = \frac{25}{4}$, $\frac{-18}{-2} = \frac{18}{2} = 9$ etc.

This explains the division of negative integers.

When one integer is divided by another non-zero integer, it is customary to write the denominator of the quotient as a positive integer.

Hence we write $\frac{7}{-2} = \frac{-7}{2}$, $\frac{-11}{-3} = \frac{11}{3}$



Now I know!

The rules of division of integers are like the rules of multiplication of integers.

- We cannot divide any number by zero.
- The quotient of two positive integers is a positive number.
- The quotient of two negative integers is a positive number.
- The quotient of a positive integer and a negative integer is always a negative number.

Practice Set 9

1. Solve :

- (i) $(-96) \div 16$ (ii) $98 \div (-28)$ (iii) $(-51) \div 68$ (iv) $38 \div (-57)$
(v) $(-85) \div 20$ (vi) $(-150) \div (-25)$ (vii) $100 \div 60$ (viii) $9 \div (-54)$
(ix) $78 \div 65$ (x) $(-5) \div (-315)$

2*. Write three divisions of integers such that the fractional form of each will be $\frac{24}{5}$.

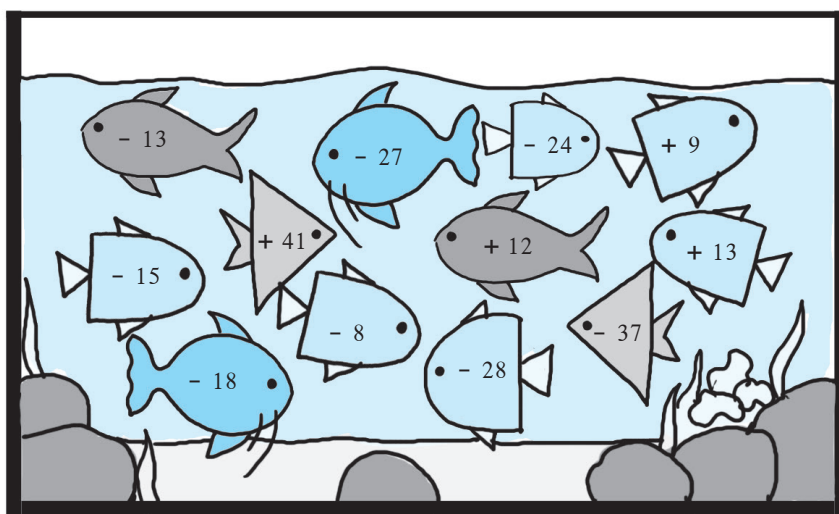
3*. Write three divisions of integers such that the fractional form of each will be $\frac{-5}{7}$.

4. The fish in the pond below, carry some numbers. Choose any 4 pairs and carry out four multiplications with those numbers. Now, choose four other pairs and carry out divisions with these numbers.

For example,

1. $(-13) \times (-15) = 195$

2. $(-24) \div 9 = \frac{-24}{9} = \frac{-8}{3}$



**Let's recall.**

- Which is the smallest prime number?
- List the prime numbers from 1 to 50. How many are they?
- Circle the prime numbers in the list below.

17, 15, 4, 3, 1, 2, 12, 23, 27, 35, 41, 43, 58, 51, 72, 79, 91, 97

Co-prime numbers : Two numbers which have only 1 as a common factor are said to be **co-prime** or **relatively prime** or **mutually prime numbers**.

For example, 10 and 21 are co-primes, because the divisors of 10 are 1, 2, 5, 10 while the divisors of 21 are 1, 3, 7, 21 and the only factor common to both 10 and 21 is 1.

Some other co-prime numbers are (3, 8) ; (4, 9) ; (21, 22) ; (22, 23) ; (23, 24).

Verify that any two consecutive natural numbers are co-primes.

**Let's learn.****Twin Prime Numbers**

If the difference between two co-prime numbers is 2, the numbers are said to be twin prime numbers.

For example : (3, 5) ; (5, 7) ; (11, 13) ; (29, 31) etc.

Practice Set 10

1. Which number is neither a prime number nor a composite number?
2. Which of the following are pairs of co-primes?
(i) 8, 14 (ii) 4, 5 (iii) 17, 19 (iv) 27, 15
3. List the prime numbers from 25 to 100 and say how many they are.
4. Write all the twin prime numbers from 51 to 100.
5. Write 5 pairs of twin prime numbers from 1 to 50.
6. Which are the even prime numbers?

**Let's learn.****Factorising a Number into its Prime Factors**

A simple but important rule given by Euclid is often used to find the GCD or HCF and LCM of numbers. The rule says that **any composite number can be written as the product of prime numbers**.

Let us learn how to find the prime factors of a number.

Example Write the number 24 in the form of the product of its prime factors.

Method for finding prime factors

Vertical arrangement

2	24
2	12
2	6
3	3
	1

Horizontal arrangement

$$\begin{aligned}
 24 &= 2 \times 12 \\
 &= 2 \times 2 \times 6 \quad \dots 12 \text{ is factorised.} \\
 &= 2 \times 2 \times 2 \times 3 \quad \dots 6 \text{ is factorised.}
 \end{aligned}$$

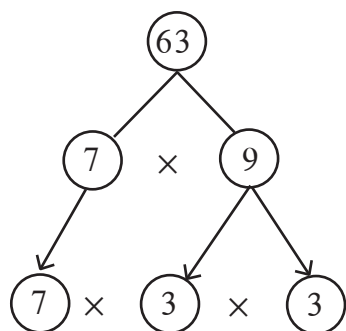
The prime factors are 2 and 3.

$$24 = 2 \times 2 \times 2 \times 3$$

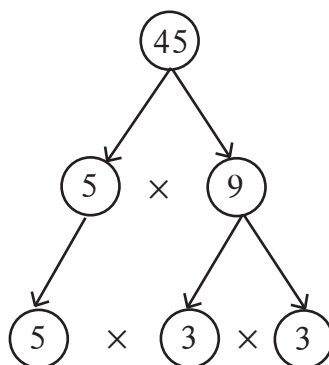
Remember :

To write a given number as a product of its prime factors is to factorise it into primes.

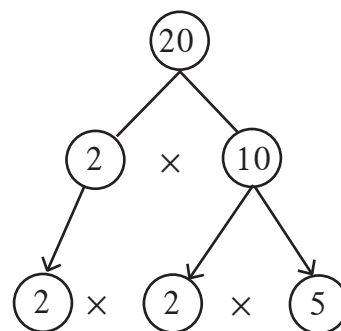
Example Write each of the given numbers as a product of its prime factors.



$$63 = 7 \times 3 \times 3$$



$$45 = 5 \times 3 \times 3$$



$$20 = 2 \times 2 \times 5$$

Example Factorise into primes: 117.

3	117
3	39
13	13
	1

$$\begin{aligned}
 117 &= 13 \times 9 \\
 &= 13 \times 3 \times 3
 \end{aligned}$$

$$117 = 3 \times 3 \times 13$$

Example Factorise into primes: 250.

2	250
5	125
5	25
5	5
	1

$$\begin{aligned}
 250 &= 2 \times 125 \\
 &= 2 \times 5 \times 25 \\
 &= 2 \times 5 \times 5 \times 5
 \end{aligned}$$

$$250 = 2 \times 5 \times 5 \times 5$$

Example Find the prime factors of 40.

Vertical arrangement

2	40
2	20
2	10
5	5
	1

$$40 = 2 \times 2 \times 2 \times 5$$

Horizontal arrangement

$$\begin{aligned} 40 &= 10 \times 4 \\ &= 5 \times 2 \times 2 \times 2 \end{aligned}$$

$$\begin{aligned} 40 &= 8 \times 5 \\ &= 2 \times 2 \times 2 \times 5 \end{aligned}$$

Practice Set 11

☉ Factorise the following numbers into primes.

(i) 32

(ii) 57

(iii) 23

(iv) 150

(v) 216

(vi) 208

(vii) 765

(viii) 342

(ix) 377

(x) 559



Let's recall.

Greatest Common Divisor (GCD) or Highest Common Factor (HCF)

We are familiar with the HCF and LCM of positive integers. Let us learn something more about them. The **HCF** or the **GCD** of given numbers is their greatest common divisor or factor.

In each of the following examples, write all the factors of the numbers and find the greatest common divisor.

(i) 28, 42

(ii) 51, 27

(iii) 25, 15, 35



Let's learn.

Prime Factors Method

It is easy to find the HCF of numbers by first factorising all the numbers.

Example Find the HCF of 24 and 32 by the prime factors method.

2	24
2	12
2	6
3	3
	1

$$\begin{aligned} 24 &= 4 \times 6 \\ &= \underline{2} \times \underline{2} \times \underline{2} \times 3 \end{aligned}$$

2	32
2	16
2	8
2	4
2	2
	1

$$\begin{aligned} 32 &= 8 \times 4 \\ &= \underline{2} \times \underline{2} \times \underline{2} \times 2 \times 2 \end{aligned}$$

The common factor 2 occurs thrice in each number. Therefore, the $HCF = 2 \times 2 \times 2 = 8$.

Example Find the HCF of 195, 312, 546.

$$195 = 5 \times 39 \\ = 5 \times \underline{3} \times \underline{13}$$

$$312 = 4 \times 78 \\ = 2 \times 2 \times 2 \times 39 \\ = 2 \times 2 \times 2 \times \underline{3} \times \underline{13}$$

$$546 = 2 \times 273 \\ = 2 \times 3 \times 91 \\ = 2 \times \underline{3} \times 7 \times \underline{13}$$

The common factors 3 and 13 each occur once in all the numbers.

$$\therefore \text{HCF} = 3 \times 13 = 39$$

Example Find the HCF of 10, 15, 12.

$$10 = 2 \times 5$$

$$15 = 3 \times 5$$

$$12 = 2 \times 2 \times 3$$

No number except 1 is a common divisor.

Hence, HCF = 1

Example Find the HCF of 60, 12, 36.

$$60 = 4 \times 15 \\ = \underline{2} \times \underline{2} \times \underline{3} \times 5$$

$$12 = 2 \times 6 \\ = \underline{2} \times \underline{2} \times \underline{3}$$

$$36 = 3 \times 12 \\ = 3 \times 3 \times 4 \\ = \underline{2} \times \underline{2} \times \underline{3} \times 3$$

$$\therefore \text{HCF} = 2 \times 2 \times 3 = 12$$

Let us work out this example in the vertical arrangement. We write all the numbers in one line and find their factors.

2	60	12	36
2	30	6	18
3	15	3	9
	5	1	3

$$\therefore \text{HCF} = 2 \times 2 \times 3 = 12$$

Note that 12 is a divisor of 36 and 60.



Now I know!

- If one of the given numbers is a divisor of all the others, then it is the HCF of the given numbers.
- If no prime number is a common divisor of all the given numbers, then 1 is their HCF because it is the only common divisor.

* Something more

2 is the HCF of any two consecutive even numbers and 1 is the HCF of any two consecutive odd numbers.

Verify the rule, by taking many different examples.

The Division Method for Finding the HCF

Example Find the HCF of 144 and 252.

$$\begin{array}{r}
 144 \overline{)252} \begin{array}{l} 1 \\ -144 \\ \hline 108 \end{array} \\
 108 \overline{)144} \begin{array}{l} 1 \\ -108 \\ \hline 36 \end{array} \\
 36 \overline{)108} \begin{array}{l} 3 \\ -108 \\ \hline 000 \end{array}
 \end{array}$$

1. Divide the bigger number by the smaller one.
 2. Divide the previous divisor by the remainder in this division.
 3. Divide the divisor of step 2 by the remainder obtained in the division in step 2.
 4. Continue like this till the remainder becomes zero. The divisor in the division in which the remainder is zero is the HCF of the given numbers.
- \therefore The HCF of 144 and 252 = 36

Example Reduce $\frac{209}{247}$ to its simplest form.

To reduce the number to its simplest form, we will find the common factors of 209 and 247. Let us find their HCF by the division method.

Here, 19 is the HCF. That is, the numerator and denominator are both divisible by 19.

$$\therefore \frac{209}{247} = \frac{209 \div 19}{247 \div 19} = \frac{11}{13}$$

$$\begin{array}{r}
 209 \overline{)247} \begin{array}{l} 1 \\ -209 \\ \hline 38 \end{array} \\
 38 \overline{)209} \begin{array}{l} 5 \\ -190 \\ \hline 19 \end{array} \\
 19 \overline{)38} \begin{array}{l} 2 \\ -38 \\ \hline 00 \end{array}
 \end{array}$$

Practice Set 12

1. Find the HCF.

(i) 25, 40

(ii) 56, 32

(iii) 40, 60, 75

(iv) 16, 27

(v) 18, 32, 48

(vi) 105, 154

(vii) 42, 45, 48

(viii) 57, 75, 102

(ix) 56, 57

(x) 777, 315, 588

2. Find the HCF by the division method and reduce to the simplest form.

(i) $\frac{275}{525}$

(ii) $\frac{76}{133}$

(iii) $\frac{161}{69}$



Let's recall.

Least Common Multiple (LCM)

The Least Common Multiple of the given numbers is the smallest number that is divisible by each of the given numbers.

- Write the tables of the given numbers and find their LCM.

(i) 6, 7

(ii) 8, 12

(iii) 5, 6, 15



Let's learn.

Example Find the LCM of 60 and 48.

Let us find the prime factors of each number.

$$60 = 2 \times 2 \times 3 \times 5$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Let us consider each prime number in these multiplications.

2 occurs a maximum of 4 times. (in the factors of 48)

3 occurs only once (in the factors of 60)

5 occurs only once (in the factors of 60)

$$\therefore \text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 10 \times 24 = 240$$

Example Find the LCM of 18, 30, 50.

$$18 = 2 \times 9$$

$$= 2 \times 3 \times 3$$

$$30 = 2 \times 15$$

$$= 2 \times 3 \times 5$$

$$50 = 2 \times 25$$

$$= 2 \times 5 \times 5$$

2, 3, 5 are the prime numbers that occur in the multiplications above.

In the products above, the number 2 occurs a maximum of times, 3 occurs a maximum of times and 5 a maximum of times.

$$\therefore \text{LCM} = 2 \times 3 \times 3 \times 5 \times 5 = 450 \quad \therefore \text{The LCM of 18, 30, 50 is 450.}$$

Example Find the LCM of 16, 28, 40.

Vertical arrangement

2	16	28	40
2	8	14	20
2	4	7	10
	2	7	5

- Use the tests of divisibility to find the prime number that divides all the numbers and then divide the given numbers. Repeat this process for the quotients as many times as possible.
- Now find the number that divides at least two of the numbers obtained and divide those numbers by the number you find. Do this as many times as possible. If a number cannot be divided, leave it as it is.
- Stop when the only common divisor you get is 1.
- Find the product of the numbers in the column on the left. Multiply this product by the numbers in the last row.

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 5 \times 7 = 560$$

Example Find the LCM and HCF of 18 and 30. Compare the product of the LCM and HCF with the product of the given numbers.

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 2 \times 3 \times 3 \times 5 = 90$$

$$\text{HCF} \times \text{LCM} = 6 \times 90 = 540$$

$$\text{Product of the two given numbers} = 18 \times 30 = 540$$

$$\text{Product of the two given numbers} = \text{HCF} \times \text{LCM}$$

2	18	30
3	9	15
	3	5

We see that the product of two numbers is equal to the product of their GCD and LCM. Verify this statement for the following pairs of numbers : (15, 48), (14, 63), (75, 120)

Example Find the LCM and HCF of 15, 45 and 105.

3	15	45	105
5	5	15	35
	1	3	7

$$15 = \underline{3} \times \underline{5}$$

$$45 = \underline{3} \times 3 \times \underline{5}$$

$$105 = \underline{3} \times \underline{5} \times 7$$

$$\text{GCD} = 3 \times 5 = 15$$

$$\text{LCM} = 3 \times 3 \times 5 \times 7 = 315$$

Example The product of two 2-digit numbers is 1280 and the GCD = 4. What is their LCM?

$$\text{GCD} \times \text{LCM} = \text{Product of given numbers}$$

$$4 \times \text{LCM} = 1280$$

$$\therefore \text{LCM} = \frac{1280}{4} = 320$$

Practice Set 13

1. Find the LCM.

- (i) 12, 15 (ii) 6, 8, 10 (iii) 18, 32 (iv) 10, 15, 20 (v) 45, 86
 (vi) 15, 30, 90 (vii) 105, 195 (viii) 12, 15, 45 (ix) 63, 81
 (x) 18, 36, 27

2. Find the HCF and LCM of the numbers given below. Verify that their product is equal to the product of the given numbers.

- (i) 32, 37 (ii) 46, 51 (iii) 15, 60 (iv) 18, 63 (v) 78, 104

The Use of LCM and HCF

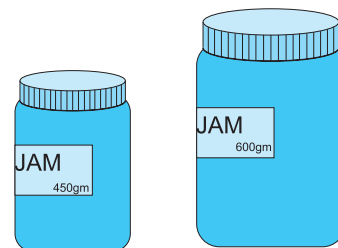
Example A shop sells a 450 g bottle of jam for 96 rupees and a bigger bottle of 600 g for 124 rupees. Which bottle is it more profitable to buy?

Solution: We have learnt the unitary method. Using that we can find the cost of 1 gm jam in each bottle and compare. However, the calculation is easier if we use a bigger common factor rather than a smaller one.

Let us use 150, the HCF of 450 and 600 to compare.

$$450 = 150 \times 3,$$

$$600 = 150 \times 4$$



∴ In the small bottle, 150 g of jam costs $\frac{96}{3} = 32$ rupees.

In the large bottle, 150 g of jam costs $\frac{124}{4} = 31$ rupees.

∴ Thus, it is more profitable to buy the 600 g bottle of jam.

Example Add $\frac{17}{28} + \frac{11}{35}$

Method 1 In order to add, let us make the denominators of the fractions equal.

Solution: $\frac{17}{28} + \frac{11}{35} = \frac{17 \times 35 + 11 \times 28}{28 \times 35} = \frac{595 + 308}{28 \times 35} = \frac{903}{28 \times 35} = \frac{903}{980} = \frac{129}{140}$

Method 2 Let us find the LCM of 28 and 35 in order to add the fractions.

$$\text{LCM} = 7 \times 4 \times 5 = 140$$

Solution: $\frac{17}{28} + \frac{11}{35} = \frac{17 \times 5}{28 \times 5} + \frac{11 \times 4}{35 \times 4} = \frac{85 + 44}{140} = \frac{129}{140}$

Taking the LCM rather than multiplying the denominators made our calculations so much easier!

Example On dividing a certain number by 8, 10, 12, 14 the remainder is always 3. Which is the smallest such number?

Solution: To find this multiple, let us find the LCM of the given divisors.

$$\text{LCM} = 2 \times 2 \times 2 \times 5 \times 3 \times 7 = 840$$

To the LCM we add the remainder obtained every time.

$$\begin{aligned} \text{Hence, that number} &= \text{LCM} + \text{remainder} \\ &= 840 + 3 = 843 \end{aligned}$$

2	8	10	12	14
2	4	5	6	7
	2	5	3	7

Example Find the LCM of the numbers 16, 20, 80.

Solution: $16 = 2 \times 2 \times 2 \times 2$

$$20 = 2 \times 2 \times 5$$

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

$$\text{LCM} = 4 \times 4 \times 5 = 80$$

Did you notice that here 80 is one of the given numbers and that the other numbers 16 and 20, are its divisors.

4	16	20	80
4	4	5	20
5	1	5	5
	1	1	1

Remember:

If the greatest of the given numbers is divisible by the other numbers, then that greatest number is the LCM of the given numbers.

In order to verify the above rule, examine these groups of numbers (18,90) (35,140,70).

Example Shreyas, Shalaka and Snehal start running from the same point on a circular track at the same time and complete one lap of the track in 16 minutes, 24 minutes and 18 minutes respectively. What is shortest period of time in which they will all reach the starting point together ?

Solution: The number of minutes they will take to reach together will be a multiple of 16, 24 and 18. To find out the smallest such number, we will find the LCM.

$$16 = 2 \times 2 \times 2 \times 2 \quad 24 = 2 \times 2 \times 2 \times 3 \quad 18 = 2 \times 3 \times 3$$

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$$

They will come together in 144 minutes or 2 hours 24 minutes.

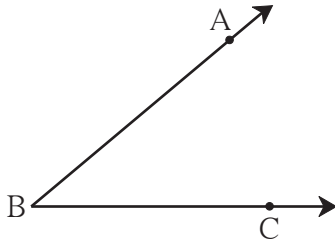
Practice Set 14

- Choose the right option.
 - The HCF of 120 and 150 is
 (1) 30 (2) 45 (3) 20 (4) 120
 - The HCF of this pair of numbers is not 1.
 (1) 13, 17 (2) 29, 20 (3) 40, 20 (4) 14, 15
- Find the HCF and LCM.
 - 14, 28 (ii) 32, 16 (iii) 17, 102, 170 (iv) 23, 69 (v) 21, 49, 84
- Find the LCM.
 - 36, 42 (ii) 15, 25, 30 (iii) 18, 42, 48 (iv) 4, 12, 20 (v) 24, 40, 80, 120
- Find the smallest number which when divided by 8, 9, 10, 15, 20 gives a remainder of 5 every time.
- Reduce the fractions $\frac{348}{319}$, $\frac{221}{247}$, $\frac{437}{551}$ to the lowest terms.
- The LCM and HCF of two numbers are 432 and 72 respectively. If one of the numbers is 216, what is the other?
- The product of two two-digit numbers is 765 and their HCF is 3. What is their LCM?
- A trader has three bundles of string 392 m, 308 m and 490 m long. What is the greatest length of string that the bundles can be cut up into without any left over string?
- * Which two consecutive even numbers have an LCM of 180?





Let's recall.



- Write the name of the angle shown alongside
- Write the name of its vertex
- Write the names of its arms
- Write the names of the points marked on its arms

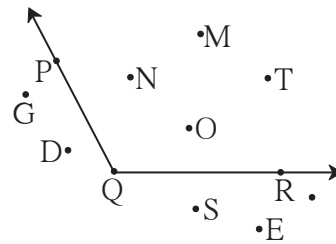


Let's learn.

The Interior and Exterior of an Angle

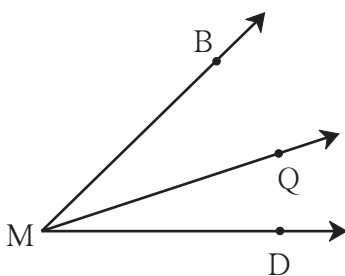
In the plane of the figure alongside, the group of points like point N, point M, point T which are not on the arms of the angle, form the **interior** of the $\angle PQR$.

The group of points in the plane of the angle like point G, point D, point E, which are neither on the arms of the angle nor in its interior, form the **exterior** of the angle.



Adjacent Angles

Look at the angles in the figure alongside. The ray MQ is a common arm of the angles $\angle BMQ$ and $\angle QMD$ while M is their common vertex. The interiors of these angles do not have a single common point. They may be said to be neighbouring angles. Such angles are called adjacent angles.



Adjacent angles have one common arm and the other arms lie on opposite sides of the common arm. They have a common vertex. Adjacent angles have separate interiors.

In the given figure, MB is the common arm of the angles $\angle BMD$ and $\angle BMQ$. However, they are not adjacent angles because they do not have separate interiors.

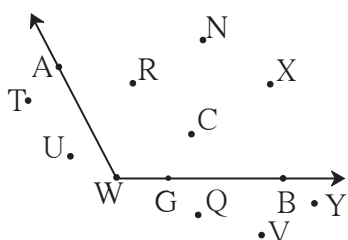


Now I know!

Two angles which have a common vertex, a common arm and separate interiors are said to be adjacent angles.

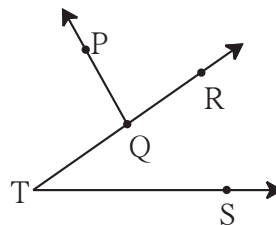
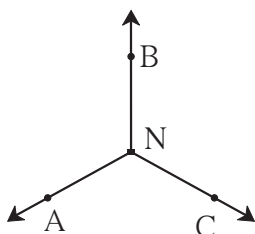
Practice Set 15

1. Observe the figure and complete the table for $\angle AWB$.



Points in the interior	
Points in the exterior	
Points on the arms of the angles	

2. Name the pairs of adjacent angles in the figures below.

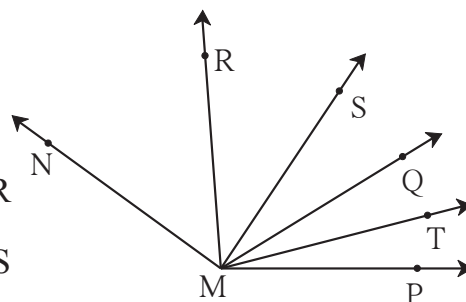


3. Are the following pairs adjacent angles?

If not, state the reason.

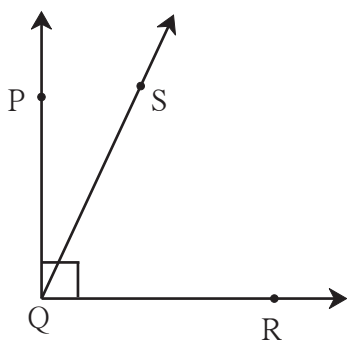
(i) $\angle PMQ$ and $\angle RMQ$ (ii) $\angle RMQ$ and $\angle SMR$

(iii) $\angle RMS$ and $\angle RMT$ (iv) $\angle SMT$ and $\angle RMS$



Let's learn.

Complementary Angles

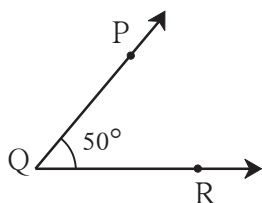
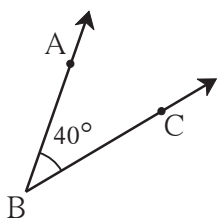


- Draw $\angle PQR$, a right angle.
- Take any point S in its interior.
- Draw ray QS.
- Add the measures of the angles $\angle PQS$ and $\angle SQR$. What will be the sum of their measures?

If the sum of the measures of two angles is 90° they are known as complementary angles.

Here, $\angle PQS$ and $\angle SQR$ are mutually complementary angles.

Example Observe the angles in the figure and enter the proper number in the box.



$$m\angle ABC = \boxed{}^\circ$$

$$m\angle PQR = \boxed{}^\circ$$

$$m\angle ABC + m\angle PQR = \boxed{}^\circ$$

The sum of the measures of $\angle ABC$ and $\angle PQR$ is 90° . Therefore, they are complementary angles.

Example Find the measure of the complement of an angle of measure 70° ?

Solution: Let the measure of the complementary angle be x .

$$70 + x = 90$$

$$\therefore 70 + x - 70 = 90 - 70$$

$$x = 20^\circ$$

The measure of the complement of an angle of measure 70° is 20° .

Example Angles of measures $(a + 15)^\circ$ and $(2a)^\circ$ are complementary. What is the measure of each angle?

$$\text{Solution: } a + 15 + 2a = 90$$

$$3a + 15 = 90$$

$$3a = 75$$

$$a = 25$$

$$\therefore a + 15 = 25 + 15 = 40^\circ$$

$$\text{and } 2a = 2 \times 25 = 50^\circ$$

Practice Set 16

- The measures of some angles are given below. Write the measures of their complementary angles.
(i) 40° (ii) 63° (iii) 45° (iv) 55° (v) 20° (vi) 90° (vii) x°
- $(y - 20)^\circ$ and $(y + 30)^\circ$ are the measures of complementary angles. Find the measure of each angle.



Let's recall.



T is a point on line AB.

- What kind of angle is $\angle ATB$?
- What is its measure?



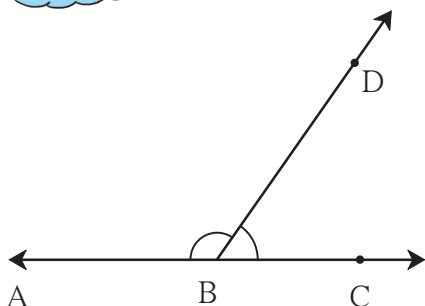
Let's learn.

Supplementary Angles

- A line AC is shown in the figure alongside. A ray BD stands on it. How many angles are formed here?

$$m\angle ABD = \boxed{}^\circ, m\angle DBC = \boxed{}^\circ$$

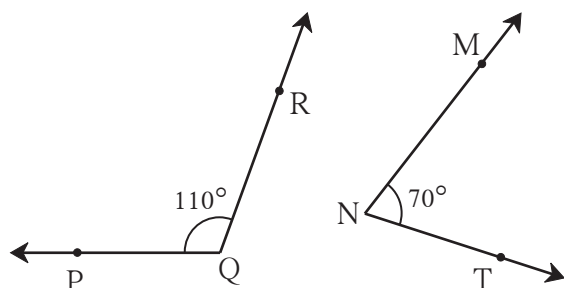
$$m\angle ABD + m\angle DBC = \boxed{}^\circ$$



If the sum of the measures of two angles is 180° they are known as supplementary angles.

Here $\angle ABD$ and $\angle DBC$ are supplementary angles.

Example Observe the angles in the figure below and enter the proper number in the box.



$$m\angle PQR = \boxed{}^\circ \quad m\angle MNT = \boxed{}^\circ$$

$$m\angle PQR + m\angle MNT = \boxed{}^\circ$$

$\angle PQR$ and $\angle MNT$ are supplementary angles.

Example Find the measure of the supplement of an angle of 135° .

Solution: Let the supplementary angle measure p° .

The sum of the measures of two supplementary angles is 180° .

$$135 + p = 180$$

$$\therefore 135 + p - 135 = 180 - 135$$

$$\therefore p = 45$$

\therefore The measure of the supplement of an angle of 135° is 45° .

Example $(a + 30)^\circ$ and $(2a)^\circ$ are the measures of two supplementary angles. What is the measure of each angle?

Solution: $a + 30 + 2a = 180$

$$\therefore 3a = 180 - 30$$

$$\therefore 3a = 150$$

$$\therefore a = 50$$

$$\therefore a + 30 = 50 + 30 = 80^\circ$$

$$\therefore 2a = 2 \times 50 = 100^\circ$$

\therefore The measures of the angles are 80° and 100° .

Practice Set 17

1. Write the measures of the supplements of the angles given below.

(i) 15° (ii) 85° (iii) 120° (iv) 37° (v) 108° (vi) 0° (vii) a°

2. The measures of some angles are given below. Use them to make pairs of complementary and supplementary angles.

$$m\angle B = 60^\circ$$

$$m\angle N = 30^\circ$$

$$m\angle Y = 90^\circ$$

$$m\angle J = 150^\circ$$

$$m\angle D = 75^\circ$$

$$m\angle E = 0^\circ$$

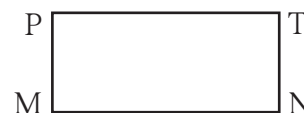
$$m\angle F = 15^\circ$$

$$m\angle G = 120^\circ$$

3. In $\triangle XYZ$, $m\angle Y = 90^\circ$. What kind of a pair do $\angle X$ and $\angle Z$ make?

4. The difference between the measures of the two angles of a complementary pair is 40° . Find the measures of the two angles.

5. $\square PTNM$ is a rectangle. Write the names of the pairs of supplementary angles.



6*. If $m\angle A = 70^\circ$, what is the measure of the supplement of the complement of $\angle A$?

7. If $\angle A$ and $\angle B$ are supplementary angles and $m\angle B = (x + 20)^\circ$, then what would be $m\angle A$?



Let's discuss.

Discuss the following statements. If a statement is right, give an example.

If it is wrong, state why.

- Two acute angles can make a pair of complementary angles.
- Two right angles can make a pair of complementary angles.
- One acute angle and one obtuse angle can make a pair of complementary angles.
- Two acute angles can form a pair of supplementary angles.
- Two right angles can form a pair of supplementary angles.
- One acute angle and one obtuse angle can form a pair of supplementary angles.



Let's learn.

Opposite Rays

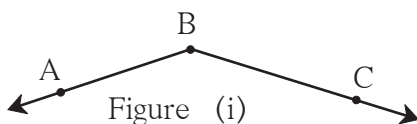


Figure (i)

- (1) Name the rays in the figure alongside.
- (2) Name the origin of the rays.
- (3) Name the angle in figure (i).

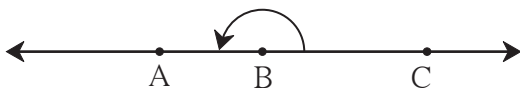


Figure (ii)

- (1) Name the angle in figure (ii) alongside.
- (2) Name the rays whose origin is point B.

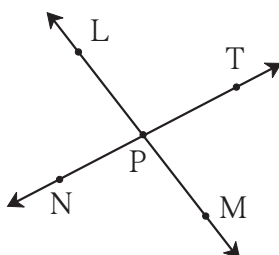
In figure (i), ray BC and ray BA meet to form an obtuse angle while in figure (ii) ray BC and ray BA meet to form a straight angle and we get a straight line. Here, ray BC and ray BA are opposite rays.



Now I know!

Two rays which have a common origin and form a straight line are said to be opposite rays.

Practice Set 18

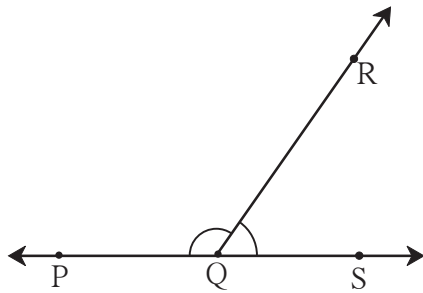


1. Name the pairs of opposite rays in the figure alongside.
2. Are the ray PM and PT opposite rays? Give reasons for your answer.



Let's learn.

Angles in a Linear Pair



- Write the names of the angles in the figure alongside.
- What type of a pair of angles is it?
- Which arms of the angles are not the common arms?
- $m\angle PQR = \boxed{}^\circ$ • $m\angle RQS = \boxed{}^\circ$
- $m\angle PQR + m\angle RQS = 180^\circ$

The angles $\angle PQR$ and $\angle RQS$ in the figure above are adjacent angles and are also supplementary angles. The arms that are not common to both angles form a pair of opposite rays i.e. these arms form a straight line. We say that these angles form a linear pair. **The sum of the measures of the angles in a linear pair is 180° .**



Now I know!

Angles which have a common arm and whose other arms form a straight line are said to be angles in a linear pair. Angles in a linear pair are supplementary angles.

Activity : Use straws or sticks to make all the kinds of angles that you have learnt about.

Practice Set 19

Draw the pairs of angles as described below. If that is not possible, say why.

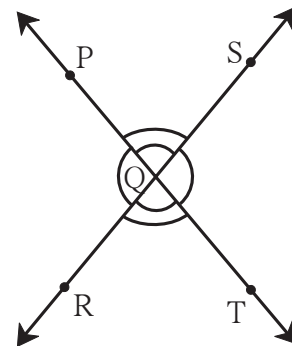
- | | |
|--|---|
| (i) Complementary angles that are not adjacent. | (ii) Angles in a linear pair which are not supplementary. |
| (iii) Complementary angles that do not form a linear pair. | (iv) Adjacent angles which are not in a linear pair. |
| (v) Angles which are neither complementary nor adjacent. | (vi) Angles in a linear pair which are complementary. |



Let's learn.

Vertically Opposite Angles

In the figure alongside, line PT and line RS intersect each other at point Q. Thus, four angles are formed. $\angle PQR$ is formed by the rays QP and QR. The rays opposite to ray QP and QR are QT and QS respectively. These opposite rays form the angle $\angle SQT$. Hence, $\angle SQT$ is called the opposite angle of $\angle PQR$.





Now I know!

The angle formed by the opposite rays of the arms of an angle is said to be its opposite angle.



Let's learn.

The Property of Vertically Opposite Angles

- Name the angle opposite to $\angle PQS$ in the figure.

As shown in the figure, $m\angle PQS = a$, $m\angle SQT = b$, $m\angle TQR = c$, $m\angle PQR = d$.

$\angle PQS$ and $\angle SQT$ are the angles in a linear pair.

$$\therefore a + b = 180^\circ$$

Also $m\angle SQT$ and $m\angle TQR$ are two angles in a linear pair.

$$\therefore b + c = 180^\circ$$

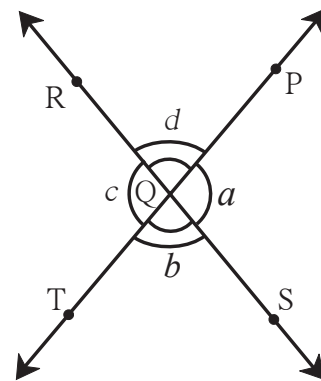
$$\therefore a + b = b + c$$

$$\therefore a = c \quad (\text{Subtracting } b \text{ from both sides.})$$

$\therefore \angle PQS$ and $\angle TQR$ are congruent angles.

Also, $m\angle PQR = m\angle SQT$

i.e. $\angle PQR$ and $\angle SQT$ are congruent angles.

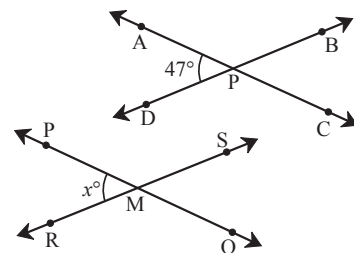


Now I know!

The vertically opposite angles formed when two lines intersect, are of equal measure.

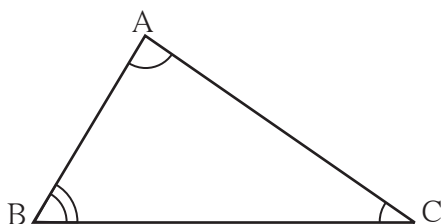
Practice Set 20

- Lines AC and BD intersect at point P. $m\angle APD = 47^\circ$
Find the measures of $\angle APB$, $\angle BPC$, $\angle CPD$.
- Lines PQ and RS intersect at point M. $m\angle PMR = x^\circ$
What are the measures of $\angle PMS$, $\angle SMQ$ and $\angle QMR$?



Let's learn.

Interior Angles of a Polygon


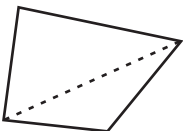
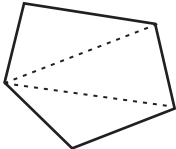
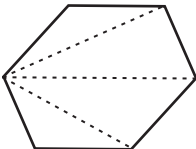
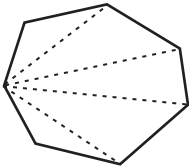


Interior Angles of a Triangle

$\angle A$, $\angle B$, $\angle C$ are the interior angles of $\triangle ABC$.

$$m\angle ABC + m\angle BAC + m\angle ACB = \boxed{}^\circ$$

Observe the table given below and draw your conclusions.

Number of sides	Name of the polygon	Polygon	Number of triangles	Sum of interior angles
3	Triangle		1	$180^\circ \times 1 = $ <input type="text"/>
4	Quadrilateral		2	$180^\circ \times 2 = $ <input type="text"/>
5	Pentagon		3	$180^\circ \times 3 = $ <input type="text"/>
6	Hexagon		4	$180^\circ \times $ <input type="text"/> $= $ <input type="text"/>
7	Heptagon		5	
8	Octagon		6	
⋮	⋮	⋮	⋮	⋮
n	A figure with n sides		(n-2)	$180^\circ \times (n-2)$

Note that the number of triangles formed in a polygon as shown above is two less than the number of sides the polygon has.



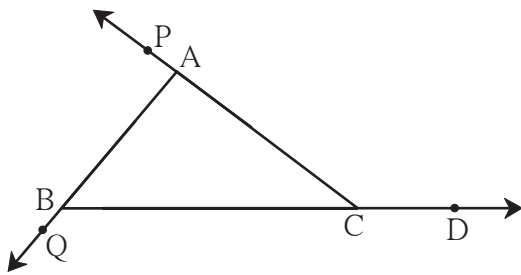
Now I know!

The sum of the measures of the interior angles of a polygon is $= 180^\circ \times (n-2)$



Let's learn.

The Exterior Angle of a Triangle



If the side BC of $\triangle ABC$ is extended as shown in the figure, an angle $\angle ACD$ is formed which lies outside the triangle.

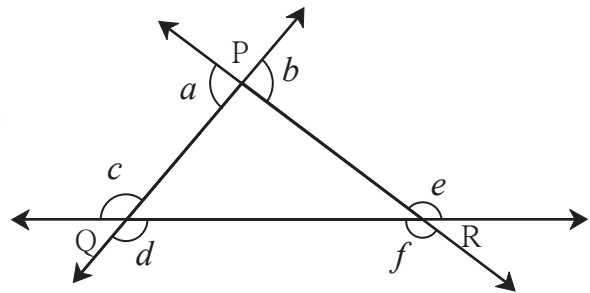
$\angle ACD$ is an exterior angle of $\triangle ABC$. $\angle ACD$ and $\angle ACB$ are angles in a linear pair. $\angle PAB$ and $\angle QBC$ are also exterior angles of $\triangle ABC$.



Now I know!

On extending one side of a triangle, the angle obtained which forms a linear pair with the adjacent interior angle of the triangle is called an exterior angle of that triangle.

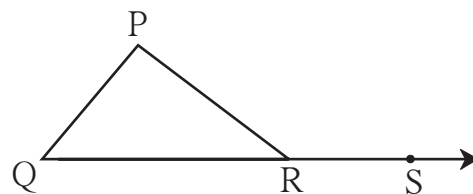
Example In the figure alongside, all exterior angles of a triangle are shown. a, b, c, d, e, f are the exterior angles of $\triangle PQR$. In the same way, every triangle has six exterior angles.



Let's learn.

The Property of an Exterior Angle of a Triangle

In the figure alongside, $\angle PRS$ is an exterior angle of $\triangle PQR$. The interior angle adjacent to it is $\angle PRQ$. The other two interior angles, $\angle P$ and $\angle Q$ are further away from $\angle PRS$. They are called the remote interior angles of $\angle PRS$.



$$m\angle P + m\angle Q + m\angle PRQ = \boxed{}^\circ \text{.....(sum of the three angles of a triangle)}$$

$$m\angle PRS + m\angle PRQ = \boxed{}^\circ \text{.....(angles in a linear pair)}$$

$$\therefore m\angle P + m\angle Q + m\angle PRQ = m\angle PRS + m\angle PRQ$$

$$\therefore m\angle P + m\angle Q = m\angle PRS \text{ (subtracting } m\angle PRQ \text{ from both sides)}$$



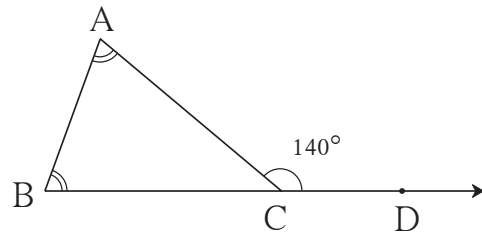
Now I know!

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

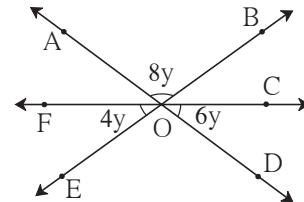
Practice Set 21

1. $\angle ACD$ is an exterior angle of $\triangle ABC$.

The measures of $\angle A$ and $\angle B$ are equal. If $m\angle ACD = 140^\circ$, find the measures of the angles $\angle A$ and $\angle B$.



2. Using the measures of the angles given in the figure alongside, find the measures of the remaining three angles.



- 3*. In the isosceles triangle ABC, $\angle A$ and $\angle B$ are equal. $\angle ACD$ is an exterior angle of $\triangle ABC$. The measures of $\angle ACB$ and $\angle ACD$ are $(3x - 17)^\circ$ and $(8x + 10)^\circ$ respectively. Find the measures of $\angle ACB$ and $\angle ACD$. Also find the measures of $\angle A$ and $\angle B$.



ICT Tools or Links

- With the help of Geogebra, draw two rays with a common origin. Using the 'Move' option, turn one ray and observe the position where the two rays become opposite rays.
- Form angles in a linear pair. By 'moving' the common arm, form many different pairs of angles all of which are linear pairs.
- Using Polygon Tools from Geogebra, draw many polygons. Verify the property of the interior angles of a polygon.





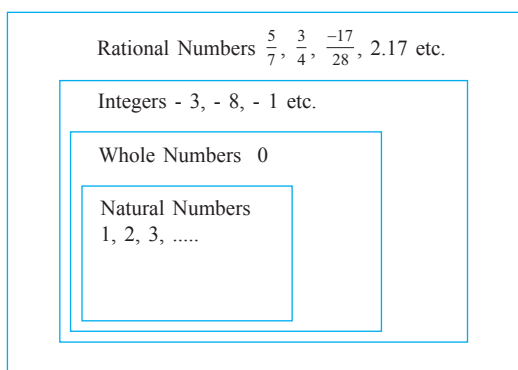
Let's learn.

Rational Numbers

In previous standards, we have learnt that the counting numbers 1, 2, 3, 4, are called natural numbers. We know that natural numbers, zero, and the opposite numbers of natural numbers together form the group of integers. We are also familiar with fractions like $\frac{7}{11}$, $\frac{2}{5}$, $\frac{1}{7}$. Is there then, a group that includes both integers and fractions? Let us see.

$4 = \frac{12}{3}$, $7 = \frac{7}{1}$, $-3 = \frac{-3}{1}$, $0 = \frac{0}{2}$ Thus, we also know that all integers can be written in the form $\frac{m}{n}$. If m is any integer and n is any non-zero integer, then the number $\frac{m}{n}$ is called a rational number.

This group of rational numbers includes all types of numbers mentioned before.



Complete the table given below.

	-3	$\frac{3}{5}$	-17	$-\frac{5}{11}$	5
Natural Number	×				✓
Integers	✓				
Rational Number	✓				

Operations on Rational Numbers

Rational numbers are written like fractions using a numerator and a denominator. That is why, operations on rational numbers are carried out as on fractions.

$$(1) \frac{5}{7} + \frac{9}{11} = \frac{55+63}{77} = \frac{118}{77}$$

$$(3) 2\frac{1}{7} + 3\frac{8}{14} = \frac{15}{7} + \frac{50}{14} \\ = \frac{30}{14} + \frac{50}{14} \\ = \frac{80}{14} = \frac{40}{7}$$

$$(2) \frac{1}{7} - \frac{3}{4} = \frac{4-21}{28} = \frac{-17}{28}$$

$$(4) \frac{9}{13} \times \frac{4}{7} = \frac{9 \times 4}{13 \times 7} = \frac{36}{91}$$

$$(5) \frac{3}{5} \times \frac{(-4)}{5} = \frac{3 \times (-4)}{5 \times 5} = \frac{-12}{25}$$

$$(6) \frac{9}{13} \times \frac{26}{3} = \frac{3 \times 2}{1} = \frac{6}{1}$$



Let's recall.

To divide one number by another is to multiply the first by the multiplicative inverse of the other.

We have seen that $\frac{5}{6}$ and $\frac{6}{5}$, $\frac{2}{11}$ and $\frac{11}{2}$ are pairs of multiplicative inverses.

Similarly, $\left(\frac{-5}{4}\right) \times \left(\frac{-4}{5}\right) = 1$; $\left(\frac{-7}{2}\right) \times \left(\frac{-2}{7}\right) = 1$ Thus $\left(\frac{-5}{4}\right)$ and $\left(\frac{-4}{5}\right)$ as also $\left(\frac{-7}{2}\right)$ and $\left(\frac{-2}{7}\right)$ are pairs of multiplicative inverses. Similarly, $\frac{-5}{4}$ and $\frac{-4}{5}$ or $\frac{-7}{2}$ and $\frac{-2}{7}$ are pairs of multiplicative inverses. That is, $\frac{-5}{4}$ and $\frac{-4}{5}$ are each other's multiplicative inverses and so are $\frac{-7}{2}$ and $\frac{-2}{7}$.



Take care!

Example The product of $\frac{-11}{9}$ and $\frac{9}{11}$ is -1 . Therefore, $\frac{-11}{9}$, $\frac{9}{11}$ is not a pair of multiplicative inverses.



Let's discuss.

Let us look at the characteristics of various groups of numbers. Discuss them in class to help you complete the following table. Consider the groups of natural numbers, integers and rational numbers. In front of each group, write the inference you make after carrying out the operations of addition, subtraction, multiplication and division, using a (✓) or a (×). Remember that you cannot divide by zero.

- If natural numbers are added, their sum is always a natural number. So, we put a tick (✓) under 'addition' in front of the group of natural numbers.
- However, if we subtract one natural number from another, the answer is not always a natural number. There are numerous examples like $7 - 10 = -3$. So, under subtraction, we put a (×). If there is a cross in the table, explain why. To explain the reason for a (×), giving only one of the numerous examples is sufficient.

Group of numbers	Addition	Subtraction	Multiplication	Division
Natural numbers	✓	× (7 - 10 = -3)	✓	× (3 ÷ 5 = $\frac{3}{5}$)
Integers				
Rational numbers				



Now I know!

- The group of natural numbers is closed for the addition and multiplication but not for the subtraction and division. In other words, the difference of any two natural numbers or the quotient obtained on dividing one natural number by another will not always be natural number.
- The group of integers is closed for addition, subtraction and multiplication but not for division.
- The group of rational numbers is closed for all operations – addition, subtraction, multiplication and division. However, we cannot divide by zero.

Practice Set 22

1. Carry out the following additions of rational numbers.

(i) $\frac{5}{36} + \frac{6}{42}$ (ii) $1\frac{2}{3} + 2\frac{4}{5}$ (iii) $\frac{11}{17} + \frac{13}{19}$ (iv) $2\frac{3}{11} + 1\frac{3}{77}$

2. Carry out the following subtractions involving rational numbers.

(i) $\frac{7}{11} - \frac{3}{7}$ (ii) $\frac{13}{36} - \frac{2}{40}$ (iii) $1\frac{2}{3} - 3\frac{5}{6}$ (iv) $4\frac{1}{2} - 3\frac{1}{3}$

3. Multiply the following rational numbers.

(i) $\frac{3}{11} \times \frac{2}{5}$ (ii) $\frac{12}{5} \times \frac{4}{15}$ (iii) $\frac{(-8)}{9} \times \frac{3}{4}$ (iv) $\frac{0}{6} \times \frac{3}{4}$

4. Write the multiplicative inverse.

(i) $\frac{2}{5}$ (ii) $\frac{-3}{8}$ (iii) $\frac{-17}{39}$ (iv) 7 (v) $-7\frac{1}{3}$

5. Carry out the divisions of rational numbers.

(i) $\frac{40}{12} \div \frac{10}{4}$ (ii) $\frac{-10}{11} \div \frac{-11}{10}$ (iii) $\frac{-7}{8} \div \frac{-3}{6}$ (iv) $\frac{2}{3} \div (-4)$
(v) $2\frac{1}{5} \div 5\frac{3}{6}$ (vi) $\frac{-5}{13} \div \frac{7}{26}$ (vii) $\frac{9}{11} \div (-8)$ (viii) $5 \div \frac{2}{5}$



Let's learn.

Numbers in between Rational Numbers

- Write all the natural numbers between 2 and 9.
- Write all the integers between -4 and 5.
- Which rational numbers are there between $\frac{1}{2}$ and $\frac{3}{4}$?

Example Let us look for rational numbers between the two rational numbers $\frac{1}{2}$ and $\frac{4}{7}$.

To do that, let us convert these numbers into fractions with equal denominators.

$$\frac{1}{2} = \frac{1 \times 7}{2 \times 7} = \frac{7}{14}, \quad \frac{4}{7} = \frac{4 \times 2}{7 \times 2} = \frac{8}{14}$$

7 and 8 are consecutive natural numbers. But, are $\frac{7}{14}$ and $\frac{8}{14}$ consecutive rational numbers?

The denominator of any number can be increased. Then the numerator also increases the same number of times.

$$\frac{7}{14} = \frac{70}{140}, \quad \frac{8}{14} = \frac{80}{140} \dots \dots \text{(Multiplying the numerator and denominator by 10)}$$

Now, $\frac{70}{140} < \frac{71}{140} \dots < \frac{79}{140} < \frac{80}{140}$ So, how many numbers do we find between $\frac{7}{14}$ and $\frac{8}{14}$?

Also, $\frac{7}{14} = \frac{700}{1400}, \quad \frac{8}{14} = \frac{800}{1400} \dots \dots \text{(Multiplying the numerator and denominator by 100)}$

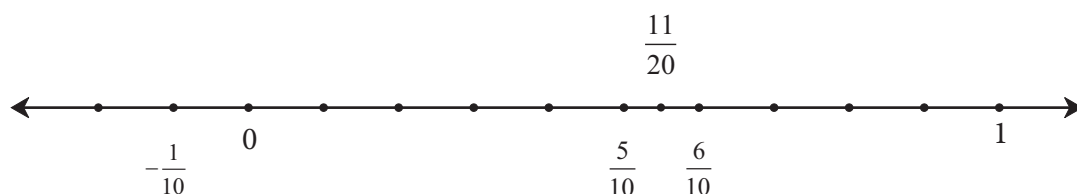
$$\text{Hence, } \frac{700}{1400} < \frac{701}{1400} \dots < \frac{799}{1400} < \frac{800}{1400}$$

Thus, when rational numbers are converted into equivalent fractions with increasingly bigger denominators, more and more rational numbers which lie between them can be expressed.

For example, let us find numbers between the rational numbers $\frac{1}{2}$ and $\frac{3}{5}$.

Let us first convert each of the numbers into their equivalent fractions.

$$\text{For example, } \frac{1}{2} = \frac{5}{10}, \quad \text{and} \quad \frac{3}{5} = \frac{6}{10}$$



On the number line, there are points showing the numbers $\frac{5}{10}$ and $\frac{6}{10}$. Let us find the mid-point of the line segment joining these two points and see what number it shows.

$$\frac{1}{2} \left(\frac{5}{10} + \frac{6}{10} \right) = \frac{11}{20} \quad \text{This is the mid-point of the line segment.}$$

$$\text{Because } \frac{6}{10} - \frac{11}{20} = \frac{12-11}{20} = \frac{1}{20} \quad \text{and also} \quad \frac{11}{20} - \frac{5}{10} = \frac{11-10}{20} = \frac{1}{20}$$

Thus, $\frac{11}{20}$ is the number that lies exactly in the middle of $\frac{5}{10}$ and $\frac{6}{10}$. It means that,

$\frac{11}{20}$ is a number that lies between $\frac{1}{2}$ and $\frac{3}{5}$. Similarly, we can find numbers that lie between $\frac{1}{2}$ and $\frac{11}{20}$ and between $\frac{11}{20}$ and $\frac{3}{5}$.



Now I know!

There are innumerable rational numbers between any two rational numbers.

Practice Set 23

⊙ Write three rational numbers that lie between the two given numbers.

(i) $\frac{2}{7}$, $\frac{6}{7}$

(ii) $\frac{4}{5}$, $\frac{2}{3}$

(iii) $-\frac{2}{3}$, $\frac{4}{5}$

(iv) $\frac{7}{9}$, $-\frac{5}{9}$

(v) $\frac{-3}{4}$, $\frac{+5}{4}$

(vi) $\frac{7}{8}$, $\frac{-5}{3}$

(vii) $\frac{5}{7}$, $\frac{11}{7}$

(viii) 0 , $\frac{-3}{4}$

★ Something more

If m is an integer, then $m + 1$ is the next bigger integer. There is no integer between m and $m + 1$. The integers that lie between any two non-consecutive integers can be counted. However, there are infinitely many rational numbers between any two rational numbers.



Let's recall.

We have learnt to multiply and divide decimal fractions.

$$\frac{35.1}{10} = 35.1 \times \frac{1}{10} = \frac{351}{10} \times \frac{1}{10} = \frac{351}{100} = 3.51$$

$$\frac{35.1}{100} = \frac{35.1}{1} \times \frac{1}{100} = \frac{351}{10} \times \frac{1}{100} = \left(\frac{351}{1000} \right) = 0.351$$

$$35.1 \times 10 = \frac{351}{10} \times 10 = 351.0$$

$$35.1 \times 1000 = \frac{351}{10} \times 1000 = \left(\frac{351000}{10} \right) = 35100.0$$

Thus we see that we can divide a decimal fraction by 100, by moving the decimal point two places to the left. To multiply by 1000, we move the point three places to the right.

The following rules are useful while multiplying and dividing.

No matter how many zeros we place at the end of the fractional part of a decimal fraction, and no matter how many zeros we place before the integral part of the number, it does not change the value of the given fraction.

$$1.35 = \frac{135}{100} \times \frac{100}{100} = \frac{13500}{10000} = 1.3500$$

$$1.35 = \frac{135}{100} \times \frac{1000}{1000} = \frac{135000}{100000} = 1.35000 \text{ etc.}$$

Also, see how we use : $1.35 = 001.35$.

$$\frac{1.35}{100} = \frac{001.35}{100} = 0.0135$$



Let's learn.

Decimal Form of Rational Numbers

Example Write the rational number $\frac{7}{4}$ in decimal form.

$$\begin{array}{r} 1.75 \\ 4 \overline{) 7.000} \\ \underline{- 4} \downarrow \\ 30 \downarrow \\ \underline{- 28} \downarrow \\ 20 \downarrow \\ \underline{- 20} \\ 00 \end{array}$$

(1) $7 = 7.0 = 7.000$ (Any number of zeros can be added after the fractional part.)

(2) 1 is the quotient and 3 the remainder after dividing 7 by 4. Now we place a decimal point after the integer 1. Writing the 0 from the dividend after the remainder 3, we divide 30 by 4. As the quotient we get now is fractional, we write 7 after the decimal point. Again we bring down the next 0 from the dividend and complete the division.

Example Write $2\frac{1}{5}$ in decimal form.

We shall find the decimal form of $2\frac{1}{5} = \frac{11}{5}$ in three different ways.

Find the decimal form of $\frac{1}{5}$.

(I)

$$\begin{array}{r} 0.2 \\ 5 \overline{) 1.0} \\ \underline{- 0} \\ 10 \\ \underline{- 10} \\ 00 \end{array}$$

$$\frac{1}{5} = 0.2$$

$$\therefore 2\frac{1}{5} = 2.2$$

(II)

$$\begin{array}{r} 2.2 \\ 5 \overline{) 11.000} \\ \underline{- 10} \\ 010 \\ \underline{- 10} \\ 00 \end{array}$$

$$\frac{11}{5} = 2.2$$

(III)

$$\begin{aligned} \frac{11}{5} &= \frac{11 \times 2}{5 \times 2} \\ &= \frac{22}{10} \\ &= 2.2 \end{aligned}$$

Example Write the rational number $\frac{-5}{8}$ in decimal form.

The decimal form of $\frac{5}{8}$ obtained by division is 0.625. $\therefore \frac{-5}{8} = -0.625$

In all the above examples, we have obtained zero as the remainder. This type of decimal form of a rational number is called the terminating decimal form.

Example Let us see how the decimal form of some rational numbers is different.

(i) Write the number $\frac{5}{3}$ in decimal form.

$$\begin{array}{r} 1.66 \\ 3 \overline{) 5.00} \\ \underline{- 3} \\ 20 \\ \underline{- 18} \\ 20 \\ \underline{- 18} \\ 2 \end{array} \quad \therefore \frac{5}{3} = 1.666\ldots$$

$$\begin{array}{r} 1.6 \\ 3 \overline{) 5.00} \\ \underline{- 3} \\ 20 \\ \underline{- 18} \\ 2 \end{array} \quad \therefore \frac{5}{3} = 1.\dot{6}$$

(ii) Write the number $\frac{2}{11}$ in decimal form.

$$\begin{array}{r} 0.18 \\ 11 \overline{) 2.00} \\ \underline{- 0} \\ 20 \\ \underline{- 11} \\ 90 \\ \underline{- 88} \\ 20 \end{array} \quad \therefore \frac{2}{11} = 0.1818\ldots$$

$$\begin{array}{r} 0.18 \\ 11 \overline{) 2.00} \\ \underline{- 0} \\ 20 \\ \underline{- 11} \\ 90 \\ \underline{- 88} \\ 20 \end{array} \quad \therefore \frac{2}{11} = 0.\overline{18}$$

(iii) Find the decimal form of $2\frac{1}{3}$. $2\frac{1}{3} = \frac{7}{3}$

$$\begin{array}{r} 2.33 \\ 3 \overline{) 7.00} \\ \underline{- 6} \\ 10 \\ \underline{- 9} \\ 10 \\ \underline{- 9} \\ 10 \\ \underline{- 9} \\ 01 \end{array} \quad 2\frac{1}{3} = 2.33\ldots$$

$$\begin{array}{r} 2.3 \\ 3 \overline{) 7.00} \\ \underline{- 6} \\ 10 \\ \underline{- 9} \\ 10 \\ \underline{- 9} \\ 01 \end{array} \quad \therefore 2\frac{1}{3} = 2.\dot{3}$$

(iv) Work out the decimal form of $\frac{5}{6}$.

$$\begin{array}{r} 0.833 \\ 6 \overline{) 5.00} \\ \underline{- 48} \\ 020 \\ \underline{- 18} \\ 020 \\ \underline{- 18} \\ 02 \end{array} \quad \frac{5}{6} = 0.833\ldots$$

$$\begin{array}{r} 0.8\dot{3} \\ 6 \overline{) 5.00} \\ \underline{- 48} \\ 020 \\ \underline{- 18} \\ 020 \\ \underline{- 18} \\ 02 \end{array} \quad \therefore \frac{5}{6} = 0.8\dot{3}$$

In all the above examples, the division does not come to an end. **Here, a single digit or a group of digits occurs repeatedly on the right of the decimal point. This type of decimal form of a rational number is called the recurring decimal form.**

If in a decimal fraction, a single digit occurs repeatedly on the right of the decimal point, we put a point above it as shown here. $2\frac{1}{3} = 2.33\ldots = 2.\dot{3}$, and if a group of digits occurs repeatedly, we show it with a horizontal line above the digits.

Thus, $\frac{2}{11} = 0.1818\ldots = 0.\overline{18}$ and $\frac{5}{6} = 0.8\dot{3}$



Now I know!

Some rational numbers have a terminating decimal form, while some have a recurring decimal form.



Let's discuss.

- Without using division, can we tell from the denominator of a fraction, whether the decimal form of the fraction will be a terminating decimal? Find out.

Practice Set 24

⊙ Write the following rational numbers in decimal form.

- (i) $\frac{13}{4}$ (ii) $\frac{-7}{8}$ (iii) $7\frac{3}{5}$ (iv) $\frac{5}{12}$ (v) $\frac{22}{7}$ (vi) $\frac{4}{3}$ (vii) $\frac{7}{9}$



Let's discuss.

An arrangement of numbers expressed using the signs of addition, subtraction, multiplication and division is called a mathematical expression.

Simplify the expression $72 \div 6 + 2 \times 2$ and find its value.

Hausa's method

$$\begin{aligned} 72 \div 6 + 2 \times 2 \\ = 12 + 2 \times 2 \\ = 12 + 4 \\ = 16 \end{aligned}$$

Mangru's Method

$$\begin{aligned} 72 \div 6 + 2 \times 2 \\ = 12 + 2 \times 2 \\ = 14 \times 2 \\ = 28 \end{aligned}$$

We got two different answers because the two children carried out the operations in different orders. To prevent this, some rules have been made which decide the order in which the operations are carried out. If these rules are followed, we will always get the same answer. When an operation is to be carried out first, it is shown using brackets in the expression.

Rules for Simplifying an Expression

- (1) If more than one operation is to be carried out, then multiplication and division are carried out first, in the order in which they occur from left to right.
- (2) After that, addition and subtraction are carried out in the order in which they occur from left to right.
- (3) If there are more than one operations in the brackets, follow the above two rules while carrying out the operations.

On applying the above rules, we see that Hausa's method is the right one.

$$\therefore 72 \div 6 + 2 \times 2 = 16$$

Let's evaluate the expressions below.

Example

$$\begin{aligned} 40 \times 10 \div 5 + 17 \\ = 400 \div 5 + 17 \\ = 80 + 17 \\ = 97 \end{aligned}$$

Example

$$\begin{aligned} 80 \div (15 + 8 - 3) + 5 \\ = 80 \div (23 - 3) + 5 \\ = 80 \div 20 + 5 \\ = 4 + 5 \\ = 9 \end{aligned}$$

Example $2 \times \{25 \times [(113 - 9) + (4 \div 2 \times 13)]\}$

$$= 2 \times \{25 \times [104 + (4 \div 2 \times 13)]\}$$

$$= 2 \times \{25 \times [104 + (2 \times 13)]\}$$

$$= 2 \times \{25 \times [104 + 26]\}$$

$$= 2 \times \{25 \times 130\}$$

$$= 2 \times 3250$$

$$= 6500$$

Example $\frac{3}{4} - \frac{5}{7} \times \frac{1}{3}$

$$= \frac{3}{4} - \frac{5}{21} \text{ (multiplication first)}$$

$$= \frac{3 \times 21 - 5 \times 4}{84} \text{ (then, subtraction)}$$

$$= \frac{63 - 20}{84}$$

$$= \frac{43}{84}$$

Remember :

Brackets may be used more than once to clearly specify the order of the operations. Different kinds of brackets such as round brackets (), square brackets [], curly brackets { }, may be used for this purpose.

When solving brackets, solve the innermost bracket first and follow it up by solving the brackets outside in turn.

Practice Set 25

⊙ Simplify the following expressions.

1. $50 \times 5 \div 2 + 24$

2. $(13 \times 4) \div 2 - 26$

3. $140 \div [(-11) \times (-3) - (-42) \div 14 - 1]$ 4. $\{(220 - 140) + [10 \times 9 + (-2 \times 5)]\} - 100$

5. $\frac{3}{5} + \frac{3}{8} \div \frac{6}{4}$

Activity : Use the signs and numbers in the boxes and form an expression such that its value will be 112.

0, 1, 2, 3, 4, 5,
6, 7, 8, 9

+ ×
÷ -

★ Something more

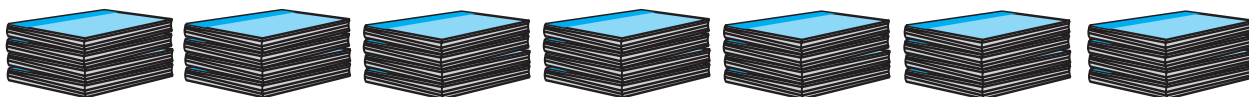
The order of the signs when simplifying an expression

B	→	O	→	D	→	M	→	A	→	S
()		×		÷		×		+		-
First, the operations in the brackets		Of multiplication e.g. $\frac{1}{4}$ of 200 $= 200 \times \frac{1}{4}$		Division		Multiplication		Addition		Subtraction

**Let's recall.**

Each of 7 children was given 4 books.

Total notebooks = $4 + 4 + 4 + 4 + 4 + 4 + 4 = 28$ notebooks



Here, addition is the operation that is carried out repeatedly.

Addition of the same number again and again can be shown as a multiplication.

Total notebooks = $4 + 4 + 4 + 4 + 4 + 4 + 4 = 4 \times 7 = 28$

**Let's learn.****Base and Index**

Let us see how the multiplication of a number by itself several times is expressed in short.

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$: Here, 2 is multiplied by itself 8 times.

This is written as 2^8 in short. This is the index form of the multiplication.

Here, 2 is called the **base** and 8, the **index** or the **exponent**.

8	← Index
2	← Base

Example $5 \times 5 \times 5 \times 5 = 5^4$ Here 5^4 is in the index form.

In the number 5^4 , 5 is the base and 4 is the index.

This is read as '5 raised to the power 4' or '5 raised to 4', or 'the 4th power of 5'.

Generally, if a is any number, $a \times a \times a \times \dots$ (m times) = a^m

Read a^m as 'a raised to the power m' or 'the mth power of a'.

Here m is a natural number.

$\therefore 5^4 = 5 \times 5 \times 5 \times 5 = 625$. Or, the value of the number $5^4 = 625$.

Similarly, $\left[\frac{-2}{3}\right]^3 = \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} = \frac{-8}{27}$ means that the value of $\left[\frac{-2}{3}\right]^3$ is $\frac{-8}{27}$.

Note that $7^1 = 7$, $10^1 = 10$. **The first power of any number is that number itself.** If the power or index of a number is 1, the convention is not to write it.

Thus $5^1 = 5$, $a^1 = a$.

Practice Set 26

1. Complete the table below.

Sr. No.	Indices (Numbers in index form)	Base	Index	Multiplication form	Value
(i)	3^4	3	4	$3 \times 3 \times 3 \times 3$	81
(ii)	16^3				
(iii)		(-8)	2		
(iv)				$\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}$	$\frac{81}{2401}$
(v)	$(-13)^4$				

2. Find the value.

(i) 2^{10}

(ii) 5^3

(iii) $(-7)^4$

(iv) $(-6)^3$

(v) 9^3

(vi) 8^1

(vii) $\left(\frac{4}{5}\right)^3$

(viii) $\left(-\frac{1}{2}\right)^4$

Square and Cube

$$3^2 = 3 \times 3$$

3^2 is read as '3 raised to 2'

or 3 'squared' or 'the square of 3'

$$5^3 = 5 \times 5 \times 5$$

5^3 is read as '5 raised to 3'

or '5 cubed' or 'the cube of 5'.

Remember :

**The second power of any number is the square of that number.
The third power of any number is the cube of that number.**



Let's learn.

Multiplication of Indices with the Same Base.

Example $2^4 \times 2^3$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 2^7$$

Therefore, $2^4 \times 2^3 = 2^{4+3} = 2^7$

Example $(-3)^2 \times (-3)^3$

$$= (-3) \times (-3) \times (-3) \times (-3) \times (-3)$$

$$= (-3)^5$$

Therefore, $(-3)^2 \times (-3)^3 = (-3)^{2+3} = (-3)^5$

Example $\left(\frac{-2}{5}\right)^2 \times \left(\frac{-2}{5}\right)^3 = \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) = \left(\frac{-2}{5}\right)^5$

Therefore, $\left(\frac{-2}{5}\right)^2 \times \left(\frac{-2}{5}\right)^3 = \left(\frac{-2}{5}\right)^{2+3} = \left(\frac{-2}{5}\right)^5$



Now I know!

If a is a rational number and m and n are positive integers, then $a^m \times a^n = a^{m+n}$

Practice Set 27

(1) Simplify.

(i) $7^4 \times 7^2$

(ii) $(-11)^5 \times (-11)^2$

(iii) $\left(\frac{6}{7}\right)^3 \times \left(\frac{6}{7}\right)^5$

(iv) $\left(-\frac{3}{2}\right)^5 \times \left(-\frac{3}{2}\right)^3$

(v) $a^{16} \times a^7$

(vi) $\left(\frac{p}{5}\right)^3 \times \left(\frac{p}{5}\right)^7$



Let's learn.

Division of Indices with the Same Base

Example $6^4 \div 6^2 = ?$

$$\frac{6^4}{6^2} = \frac{6 \times 6 \times 6 \times 6}{6 \times 6}$$

$$= 6 \times 6$$

$$= 6^2$$

$$\therefore 6^4 \div 6^2 = 6^{4-2} = 6^2$$

Example $(-2)^5 \div (-2)^3 = ?$

$$\frac{(-2)^5}{(-2)^3} = \frac{(-2) \times (-2) \times (-2) \times (-2) \times (-2)}{(-2) \times (-2) \times (-2)}$$

$$= (-2)^2$$

$$\therefore (-2)^5 \div (-2)^3 = (-2)^2$$



Now I know!

If a is a non-zero rational number, m and n are positive integers and $m > n$, then $\frac{a^m}{a^n} = a^{m-n}$

The meaning of a^0

If $a \neq 0$

Then $\frac{a^m}{a^m} = 1$

Also, $\frac{a^m}{a^m} = a^{m-m} = a^0$

$$\therefore \boxed{a^0 = 1}$$

The meaning of a^{-m}

$$a^{-m} = a^{-m} \times 1$$

$$= a^{-m} \times \frac{a^m}{a^m}$$

$$= \frac{a^{-m+m}}{a^m}$$

$$= \frac{a^0}{a^m} = \frac{1}{a^m}$$

$$\boxed{a^{-m} = \frac{1}{a^m}}$$

$$a^{-m} = \frac{1}{a^m} \quad \therefore a^{-1} = \frac{1}{a}$$

$$a \times \frac{1}{a} = 1, \quad \therefore a \times a^{-1} = 1$$

$\therefore a^{-1}$ is the multiplicative inverse of a .

Thus, the multiplicative inverse

of $\frac{5}{3}$ is $\frac{3}{5}$.

$$\therefore \boxed{\left(\frac{5}{3}\right)^{-1} = \frac{3}{5}}$$

Example Let us consider $\left(\frac{4}{7}\right)^{-3} \cdot \left(\frac{4}{7}\right)^{-3} = \frac{1}{\frac{4}{7} \times \frac{4}{7} \times \frac{4}{7}} = \frac{1}{\frac{64}{343}} = \frac{343}{64} = \left(\frac{7}{4}\right)^3$



Now I know!

Hence, we get the rule that if $a \neq 0$, $b \neq 0$ and m is a positive integer, then $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Let us see what rule we get by observing the following examples :

Example $(3)^4 \div (3)^6$

$$\begin{aligned} &= \frac{3^4}{3^6} \\ &= \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{1}{3^2} \\ 3^4 \div 3^6 &= 3^{4-6} = 3^{-2} \end{aligned}$$

Example $\left(\frac{3}{5}\right)^2 \div \left(\frac{3}{5}\right)^5$

$$\begin{aligned} &= \frac{\frac{3}{5} \times \frac{3}{5}}{\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}} = \frac{1}{\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}} = \frac{1}{\left(\frac{3}{5}\right)^3} \\ \therefore \left(\frac{3}{5}\right)^2 \div \left(\frac{3}{5}\right)^5 &= \left(\frac{3}{5}\right)^{2-5} = \left(\frac{3}{5}\right)^{-3} \end{aligned}$$



Now I know!

If a is a rational number, $a \neq 0$ and m and n are positive integers, then $\frac{a^m}{a^n} = a^{m-n}$



Let's learn.

Observe what happens if the base is (-1) and the index is a whole number.

$$(-1)^6 = \underbrace{(-1) \times (-1)}_{1} \times \underbrace{(-1) \times (-1)}_{1} \times \underbrace{(-1) \times (-1)}_{1} = 1 \times 1 \times 1 = 1$$

$$(-1)^5 = \underbrace{(-1) \times (-1)}_{1} \times \underbrace{(-1) \times (-1)}_{1} \times (-1) = 1 \times 1 \times (-1) = -1$$

If m is an even number then $(-1)^m = 1$, and if m is an odd number, then $(-1)^m = -1$

Practice Set 28

1. Simplify.

(i) $a^6 \div a^4$

(ii) $m^5 \div m^8$

(iii) $p^3 \div p^{13}$

(iv) $x^{10} \div x^{10}$

2. Find the value.

(i) $(-7)^{12} \div (-7)^{12}$

(ii) $7^5 \div 7^3$

(iii) $\left(\frac{4}{5}\right)^3 \div \left(\frac{4}{5}\right)^2$

(iv) $4^7 \div 4^5$



Let's learn.

The Index of the Product or Quotient of Two Numbers

Let us observe the following examples to see what rule we get.

Example $(2 \times 3)^4$

$$\begin{aligned}
 &= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \\
 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^4 \times 3^4
 \end{aligned}$$

Example $\left(\frac{4}{5}\right)^3$

$$\begin{aligned}
 &= \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \\
 &= \frac{4 \times 4 \times 4}{5 \times 5 \times 5} = \frac{4^3}{5^3}
 \end{aligned}$$



Now I know!

If a and b are non-zero rational numbers and m is an integer, then

$$(1) (a \times b)^m = a^m \times b^m \quad (2) \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$(a^m)^n$, that is, the Power of a Number in Index Form

Example $(5^2)^3$

$$\begin{aligned}
 &= 5^2 \times 5^2 \times 5^2 \\
 &= 5^{2+2+2} \\
 &= 5^{2 \times 3} \\
 &= 5^6
 \end{aligned}$$

Example $(7^{-2})^{-5} = \frac{1}{(7^{-2})^5}$

$$\begin{aligned}
 &= \frac{1}{7^{-2} \times 7^{-2} \times 7^{-2} \times 7^{-2} \times 7^{-2}} \\
 &= \frac{1}{7^{(-2) \times 5}} \\
 &= \frac{1}{7^{-10}} = 7^{10}
 \end{aligned}$$

$$a^{-m} = \frac{1}{a^m}$$

Example $\left(\left(\frac{2}{5}\right)^{-2}\right)^3$

$$= \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} = \left(\frac{2}{5}\right)^{(-2)+(-2)+(-2)} = \left(\frac{2}{5}\right)^{-6}$$

$$(a^m)^n = a^m \times a^m \times a^m \times \dots \times a^m \text{ } n \text{ times} = a^{m+m+m \dots m \text{ } n \text{ times}} = a^{m \times n}$$

From the above examples, we get the following rule.



Now I know!

If a is a non-zero rational number and m and n are integers, then $(a^m)^n = a^{m \times n} = a^{mn}$

Remember :

$$a^m \times a^n = a^{m+n}, \quad a^m \div a^n = a^{m-n}, \quad a^1 = a, \quad a^0 = 1, \quad a^{-m} = \frac{1}{a^m},$$

$$(ab)^m = a^m \times b^m, \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \quad (a^m)^n = a^{mn}, \quad \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

1. Simplify.

$$\begin{array}{lllll} \text{(i)} \left[\left(\frac{15}{12} \right)^3 \right]^4 & \text{(ii)} (3^4)^{-2} & \text{(iii)} \left(\left(\frac{1}{7} \right)^{-3} \right)^4 & \text{(iv)} \left(\left(\frac{2}{5} \right)^{-2} \right)^{-3} & \text{(v)} (6^5)^4 \\ \text{(vi)} \left[\left(\frac{6}{7} \right)^5 \right]^2 & \text{(vii)} \left[\left(\frac{2}{3} \right)^{-4} \right]^5 & \text{(viii)} \left[\left(\frac{5}{8} \right)^3 \right]^{-2} & \text{(ix)} \left[\left(\frac{3}{4} \right)^6 \right]^1 & \text{(x)} \left[\left(\frac{2}{5} \right)^{-3} \right]^2 \end{array}$$

(i) $\left(\frac{2}{7}\right)^{-2}$ (ii) $\left(\frac{11}{3}\right)^{-5}$ (iii) $\left(\frac{1}{6}\right)^{-3}$ (iv) $(y)^4$



10000000000000000000000000000000

**Let's recall.****Finding the square root of a perfect square**

When a number is multiplied by itself the product obtained is the square of the number.

Example $6 \times 6 = 6^2 = 36$

$6^2 = 36$ is read as 'The square of 6 is 36.'

Example $(-5) \times (-5) = (-5)^2 = 25$

$(-5)^2 = 25$ is read as 'The square of (-5) is 25.'

**Let's learn.****★ Finding the square root of a given number**

Example $3 \times 3 = 3^2 = 9$ Here, the square of 3 is 9.

Or, we can say that the square root of 9 is 3.

The symbol $\sqrt{\quad}$ is used for 'square root'.

$\sqrt{9}$ means the square root of 9. $\therefore \sqrt{9} = 3$

Example $7 \times 7 = 7^2 = 49$ $\therefore \sqrt{49} = 7$

Example $8 \times 8 = 8^2 = 64$. Hence $\sqrt{64} = 8$

$(-8) \times (-8) = (-8)^2 = 64$. Hence, $\sqrt{64} = -8$.

Thus, if x is a positive number, it has two square roots.

Of these, the negative square root is shown as $-\sqrt{x}$ and

the positive one as \sqrt{x} .

Example Find the square root of 81.

$81 = 9 \times 9 = -9 \times -9$ $\therefore \sqrt{81} = 9$ and $-\sqrt{81} = -9$

Mostly, we consider the positive square root.

★ Finding the square root by the factors method

Example Find the square root of 144.

Find the prime factors of the given number and put them in pairs of equal numbers.

$$144 = 2 \times 72$$

$$= 2 \times 2 \times 36$$

$$= 2 \times 2 \times 2 \times 18$$

$$= \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$$

Form pairs of equal factors from the prime factors obtained.

Take one factor from each pair and multiply.

$$\sqrt{144} = 2 \times 2 \times 3 = 12 \quad \therefore \sqrt{144} = 12$$

2	144
2	72
2	36
2	18
3	9
3	3
	1

Example Find the square root of 324.

Find the prime factors of the given number and put them in pairs of equal factors.

$$\begin{aligned} 324 &= 2 \times 162 \\ &= 2 \times 2 \times 81 \\ &= 2 \times 2 \times 3 \times 27 \\ &= 2 \times 2 \times 3 \times 3 \times 9 \\ &= \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \end{aligned}$$

To find the square root, take one number from each pair and multiply.

$$\sqrt{324} = 2 \times 3 \times 3 = 18$$

$$\therefore \sqrt{324} = 18$$

2	324
2	162
3	81
3	27
3	9
3	3
	1

Practice Set 30

⊙ Find the square root. (i) 625 (ii) 1225 (iii) 289 (iv) 4096 (v) 1089

* Something more (Square root by the division method)

Find the square root of :

(1) 9801

	99
9	<u>9801</u>
+ 9	<u>81</u>
189	<u>1701</u>
+ 9	<u>1701</u>
198	0000

$$\sqrt{9801} = 99$$

(2) 19321

	139
1	<u>19321</u>
+ 1	<u>1</u>
23	<u>093</u>
+ 3	<u>69</u>
269	<u>2421</u>
+ 9	<u>2421</u>
278	0000

(3) 141.61

	11.9
1	<u>141.61</u>
+ 1	<u>1</u>
21	<u>041</u>
+ 1	<u>21</u>
229	<u>2061</u>
+ 9	<u>2061</u>
238	0000

This method can be used to find the square root of numbers which have many prime factors and are, therefore, difficult to factorise.

Now let us take $\sqrt{137}$ to see one more use.

	11.7
1	<u>137.00</u>
+ 1	<u>1</u>
21	<u>037</u>
+ 1	<u>21</u>
227	<u>1600</u>
+ 7	<u>1589</u>
234	11

$$\sqrt{137} > 11.7$$

$$\text{But } (11.8)^2 = 139.24$$

$$\therefore 11.7 < \sqrt{137} < 11.8$$

Thus, we can find the approximate value of $\sqrt{137}$. This method can be used to find the approximate square root of a number whose square root is not a whole number.



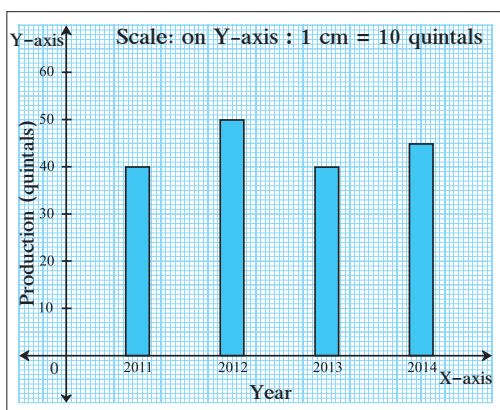


Let's discuss.

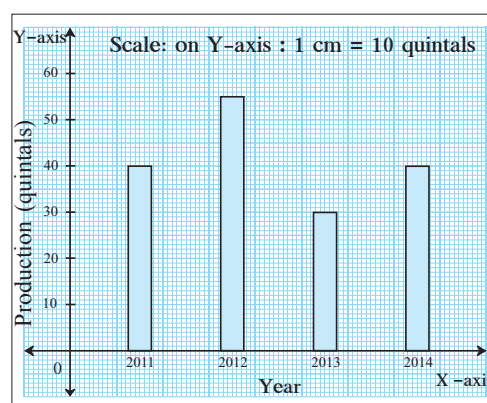
Joint Bar Graph

Observe the two bar graphs below which show the wheat production in quintals in Ajay's and Vijay's farms.

Ajay's wheat production

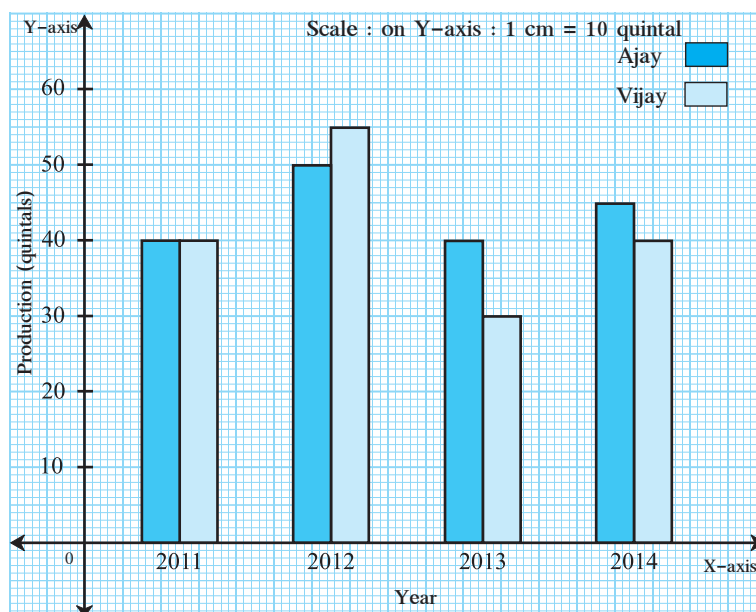


Vijay's wheat production



Let us see if we can show the information from both graphs in a single graph. Look at the graph below. In this way, more information can be given using less space. Besides, comparing Ajay and Vijay's wheat production also becomes easier. Such graphs are called joint bar graphs.

Ajay and Vijay's wheat production

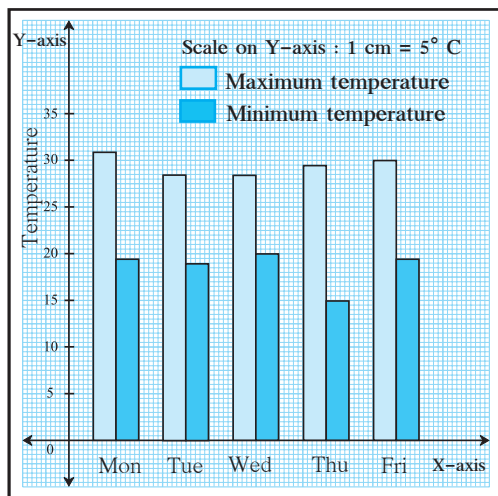


Observe the graph shown alongside and answer the following questions.

- In which year did they both produce equal quantities of wheat ?
- In year 2014, who produced more wheat?
- In year 2013, how much wheat did Ajay and Vijay each produce?

Reading a Joint Bar Graph

The minimum and maximum temperature in Pune for five days is given. Read the joint bar graph and answer the questions below.



- What data is shown on X-axis?
- What data is shown on Y-axis?
- Which day had the highest temperature?
- On which day is the minimum temperature the highest?
- On Thursday, what is the difference between the minimum and maximum temperature?
- On which day is the difference between the minimum and maximum temperature the greatest ?



Let's learn.

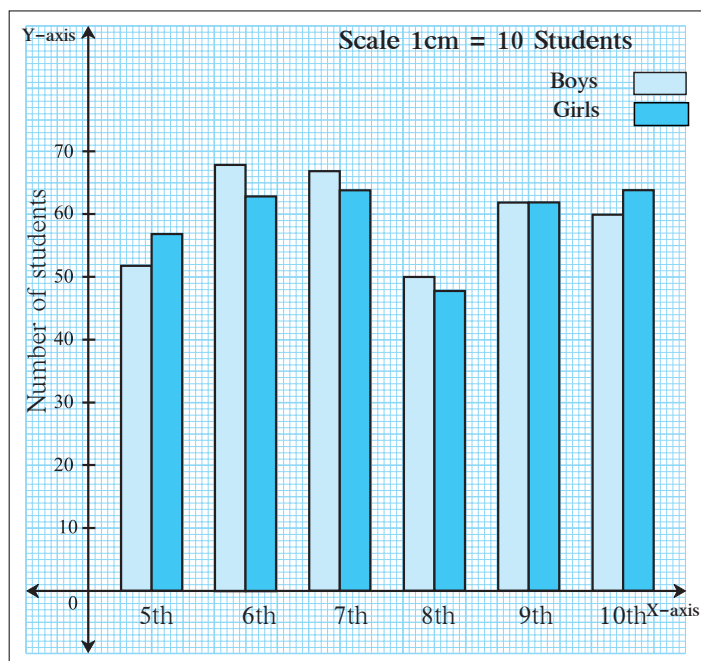
Drawing a Joint Bar Graph

The number of boys and girls in a school is given. Draw a joint bar graph to show this information.

Class	5th	6th	7th	8th	9th	10th
Boys	52	68	67	50	62	60
Girls	57	63	64	48	62	64

Steps for drawing a Joint Bar Graph

1. On a graph paper, draw the X-axis and Y-axis and their point of intersection.
2. Keeping the distance between two sets of joint bars equal, show the classes on X-axis.
3. Choose a scale for the Y-axis.
For example, 1 unit = 10 girls/boys. Mark the numbers of boys and girls on the Y-axis.
4. Using the scale, work out the height of columns required to show the numbers of boys and girls in each class. Use different colours to show the different bars in each set.





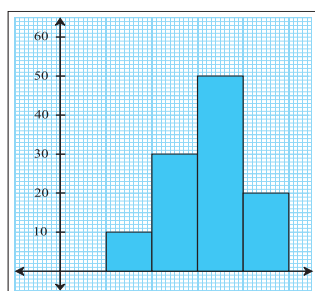
Now I know!

- In a joint bar graph, the width of all columns should be equal.
- The distance between any two consecutive sets of joint bars should be equal.
- A joint bar graph is used for a comparative study.

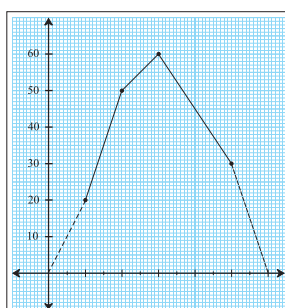


My friend, Maths : newspapers, magazines, presentation of data.

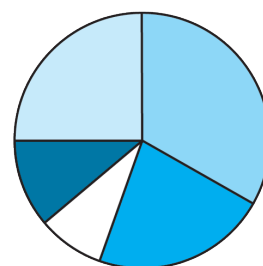
- Collect various kinds of graphs from newspapers and discuss them.



1. Histogram



2. Line Graph



3. Pie Chart



ICT Tools or Links

When presenting data, different kinds of graphs are used instead of only bar graphs. With the help of your teacher, take a look at the various kinds of graphs seen in MS-Excel, Graph Matica, Geogebra.

Practice Set 31

- The number of saplings planted by schools on World Tree Day is given in the table below. Draw a joint bar graph to show these figures.

School Name \ Name of sapling	Almond	Karanj	Neem	Ashok	Gulmohar
Nutan Vidyalaya	40	60	72	15	42
Bharat Vidyalaya	42	38	60	25	40

- The table below shows the number of people who had the different juices at a juice bar on a Saturday and a Sunday. Draw a joint bar graph for this data.

Days \ Fruits	Sweet Lime	Orange	Apple	Pineapple
Saturday	43	30	56	40
Sunday	59	65	78	67

3. The following numbers of votes were cast at 5 polling booths during the Gram Panchayat elections. Draw a joint bar graph for this data.

Persons \ Booth No.	1	2	3	4	5
Men	200	270	560	820	850
Women	700	240	340	640	470

4. The maximum and minimum temperatures of five Indian cities are given in °C. Draw a joint bar graph for this data.

Temperature \ City	Delhi	Mumbai	Kolkata	Nagpur	Kapurthala
Maximum temperature	35	32	37	41	37
Minimum temperature	26	25	26	29	26

5. The numbers of children vaccinated in one day at the government hospitals in Solapur and Pune are given in the table. Draw a joint bar graph for this data.

City \ Vaccine	D.P.T. (Booster)	Polio (Booster)	Measles	Hepatitis
Solapur	65	60	65	63
Pune	89	87	88	86

6. The percentage of literate people in the states of Maharashtra and Gujarat are given below. Draw a joint bar graph for this data.

State \ Year	1971	1981	1991	2001	2011
Maharashtra	46	57	65	77	83
Gujarat	40	45	61	69	79

A joint bar graph is useful for drawing conclusions from observations recorded in a science experiment as well in geography and economics.

Maths is fun!

$$1 + 3 = 2^2$$

$$1 + 3 + 5 = 3^2$$

$$1 + 3 + 5 + 7 = 4^2$$

Can you obtain the formula $1 + 3 + \dots + (2n - 1) = n^2$?

Verify this formula for $n = 6, 7, 8, \dots$





Let's learn.

Algebraic Expressions

Look at the arrangements of sticks given below and observe the pattern.

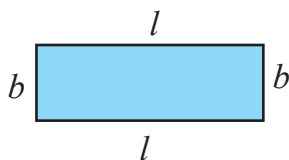
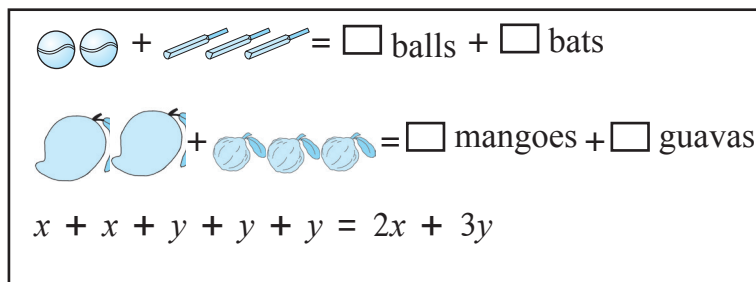
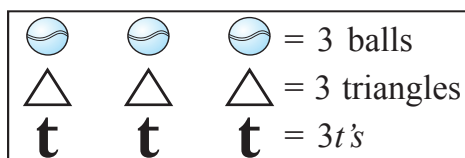
Arrangements of sticks			
Squares	1	2	3	4	..	10	..	n
Number of sticks	4	7	10	13
	$3 + 1$	$6 + 1$	$9 + 1$	$12 + 1$
	$3 \times 1 + 1$	$3 \times 2 + 1$	$3 \times 3 + 1$	$3 \times 4 + 1$		$3 \times 10 + 1$		$3 \times n + 1$

On observing the pattern above, we notice that

Number of sticks = $3 \times$ number of squares + 1

Here, the number of squares changes. It could be any of the numbers 2, 3, 4, ... , 10, ... If we do not know the number of squares, we write a letter in its place. Here, the number of squares is shown by the letter n .

' n ' is a variable. $3 \times n + 1$ which is the same as $3n + 1$ is an algebraic expression in the variable n .



$$\begin{aligned} \text{Perimeter of a rectangle} &= 2l + 2b \\ &= 2(l + b) \end{aligned}$$



Now I know!

$3n + 1$, $3t$, $2x + 3y$, $2(l + b)$ are algebraic expressions.

n , t , y , l , b , x are the variables in these expressions.



Let's learn.

In the expression $3x$, 3 is the coefficient of the variable x .

In the expression $-15t$, -15 is the coefficient of the variable t .

An expression in which multiplication is the only operation is called a 'term'.

An algebraic expression may have one term or may be the sum of several terms.

Term	Coefficient	Variables
$11mn$	11	m, n
$-9x^2y^3$	-9	x, y
$\frac{5}{6}p$	$\frac{5}{6}$	p
a	1	a

Example In the algebraic expression $4x^2 - 2y + \frac{5}{6}xz$, $4x^2$ is the first term and

4 is the coefficient in it.

$-2y$ is the second term with the coefficient 2.

$\frac{5}{6}xz$ is the third term and $\frac{5}{6}$ is the coefficient in it.

Remember :

- The algebraic expression $15 - x$ has two terms. The first term 15 is a number. The second term is $-x$. Here, the coefficient of the variable x is (-1) .
- Terms which have the same variables with the same powers are called 'like terms'.

Like terms

(i) $2x, 5x, -\frac{2}{3}x$, (ii) $-5x^2y, \frac{6}{7}yx^2$

Unlike terms

(i) $7xy, 9y^2, -2xyz$, (ii) $8mn, 8m^2n^2, 8m^3n$

Types of Algebraic Expressions

Expressions are named after the number of terms they have. Expressions with one term are called monomials, those with two terms, binomials, with three terms, trinomials and if they have more than three terms, they are called polynomials.

Monomials

- $4x$
- $\frac{5}{6}m$
- -7

Binomials

- $2x - 3y$
- $2l + 2b$
- $3mn - 5m^2n$

Trinomials

- $a + b + c$
- $x^2 - 5x + 6$
- $8a^3 - 5a^2b + c$

Polynomials

- $a^3 - 3a^2b + 3ab - b^3$
- $4x^4 - 7x^2 + 9 - 5x^3 - 16x$
- $5x^5 - \frac{1}{2}x + 8x^3 - 5$

Practice Set 32

- ⊙ Classify the following algebraic expressions as monomials, binomials, trinomials or polynomials.

(i) $7x$

(ii) $5y - 7z$

(iii) $3x^3 - 5x^2 - 11$

(iv) $1 - 8a - 7a^2 - 7a^3$

(v) $5m - 3$

(vi) a

(vii) 4

(viii) $3y^2 - 7y + 5$



Let's learn.

Addition of Algebraic Expressions

★ Addition of monomials

Example $3 \text{ guavas} + 4 \text{ guavas} = (3 + 4) \text{ guavas} = 7 \text{ guavas}$

Example $3x + 4x = (3 + 4)x = 7x$

Like terms are added as we would add up things of the same kind.

Example Add.

(i) $-3x - 8x + 5x = (-3 - 8 + 5)x = -6x$

(ii) $\frac{2}{3}ab - \frac{5}{7}ab = (\frac{2}{3} - \frac{5}{7})ab = \frac{-1}{21}ab$

(iii) $-2p^2 + 7p^2 = (-2 + 7)p^2 = 5p^2$

Think about it.

$3x + 4y =$ How many?

$3 \text{ guavas} + 4 \text{ mangoes} = 7 \text{ guavas?}$

$7m - 2n = 5m ?$

★ Addition of binomial expressions

Horizontal arrangement

Example $(2x + 4y) + (3x + 2y)$

$$= 2x + 3x + 4y + 2y$$

$$= 5x + 6y$$

Vertical arrangement

$$\begin{array}{r} 2x + 4y \\ + \quad 3x + 2y \\ \hline 5x + 6y \end{array}$$

To add like terms, we add their coefficients and write the variable after their sum.

Example Add. $9x^2y^2 - 7xy$; $3x^2y^2 + 4xy$

Horizontal arrangement

$$(9x^2y^2 - 7xy) + (3x^2y^2 + 4xy)$$

$$= 9x^2y^2 - 7xy + 3x^2y^2 + 4xy$$

$$= (9x^2y^2 + 3x^2y^2) + (-7xy + 4xy)$$

$$= 12x^2y^2 - 3xy$$

Vertical arrangement

$$\begin{array}{r} 9x^2y^2 - 7xy \\ + \quad 3x^2y^2 + 4xy \\ \hline 12x^2y^2 - 3xy \end{array}$$



Take care!

In $3x + 7y$, the two terms are not like terms. Hence, their sum can only be written as $3x + 7y$ or as $7y + 3x$.

Practice Set 33

⊙ Add.

(i) $9p + 16q$; $13p + 2q$

(ii) $2a + 6b + 8c$; $16a + 13c + 18b$

(iii) $13x^2 - 12y^2$; $6x^2 - 8y^2$

(iv) $17a^2b^2 + 16c$; $28c - 28a^2b^2$

(v) $3y^2 - 10y + 16$; $2y - 7$

(vi) $-3y^2 + 10y - 16$; $7y^2 + 8$

**Let's learn.****Subtraction of Algebraic Expressions**

We have learnt that to subtract one integer from another is to add its opposite integer to the other.

We shall use the same rule for subtraction of algebraic expressions.

Example $18 - 7$
 $= 18 + (-7) = 11$

Example $9x - 4x$
 $= [9 + (-4)]x = 5x$

Example Subtract the second expression from the first.

$$16x + 23y + 12z ; 9x - 27y + 14z$$

Horizontal arrangement

$$\begin{aligned} & (16x + 23y + 12z) - (9x - 27y + 14z) \\ &= 16x + 23y + 12z - 9x + 27y - 14z \\ &= (16x - 9x) + (23y + 27y) + (12z - 14z) \\ &= 7x + 50y - 2z \end{aligned}$$

Vertical arrangement

$$\begin{array}{r} 16x + 23y + 12z \\ - 9x - 27y + 14z \\ \hline 7x + 50y - 2z \end{array}$$

(Change the sign of every term in the expression to be subtracted and then add the two expressions.)

Practice Set 34

⊙ Subtract the second expression from the first.

(i) $(4xy - 9z) ; (3xy - 16z)$ (ii) $(5x + 4y + 7z) ; (x + 2y + 3z)$

(iii) $(14x^2 + 8xy + 3y^2) ; (26x^2 - 8xy - 17y^2)$ (iv) $(6x^2 + 7xy + 16y^2) ; (16x^2 - 17xy)$

(v) $(4x + 16z) ; (19y - 14z + 16x)$

**Let's learn.****Multiplication of Algebraic Expressions***** Multiplying a monomial by a monomial**

Example $3x \times 12y$
 $= 3 \times 12 \times x \times y$
 $= 36xy$

Example $(-12x) \times 3y^2$
 $= -12 \times 3 \times x \times y \times y$
 $= -36xy^2$

Example $2a^2 \times 3ab^2$
 $= 2 \times 3 \times a^2 \times a \times b^2$
 $= 6a^3 b^2$

Example $(-3x^2) \times (-4xy)$
 $= (-3) \times (-4) \times x^2 \times x \times y$
 $= 12x^3 y$

When multiplying two monomials, first multiply the coefficients along with the signs. Then multiply the variables.

★ **Multiplying a binomial by a monomial**

Example $x(x + y)$
 $= x \times x + x \times y$
 $= x^2 + xy$

Example $(7x - 6y) \times 3z = 7x \times 3z - 6y \times 3z$
 $= 7 \times 3 \times x \times z - 6 \times 3 \times y \times z$
 $= 21xz - 18yz$

★ **Multiplying a binomial by a binomial**

Example

$$\begin{array}{r} 3x + 4y \\ \times 5x + 7y \\ \hline 15x^2 + 20xy \quad \text{[Multiplying by } 5x\text{]} \\ + 21xy + 28y^2 \quad \text{[Multiplying by } 7y\text{]} \\ \hline 15x^2 + 41xy + 28y^2 \quad \text{[adding]} \end{array}$$

$$\begin{aligned} &(3x + 4y)(5x + 7y) \\ &= 3x(5x + 7y) + 4y(5x + 7y) \\ &= 3x \times 5x + 3x \times 7y + 4y \times 5x + 4y \times 7y \\ &= 15x^2 + 21xy + 20xy + 28y^2 \\ &= 15x^2 + 41xy + 28y^2 \end{aligned}$$

Example Find the area of a rectangular field whose length is $(2x + 7)$ m and breadth is $(x + 2)$ m.

Solution: Area of rectangular field = length \times breadth = $(2x + 7) \times (x + 2)$
 $= 2x(x + 2) + 7(x + 2)$
 $= 2x^2 + 11x + 14$

Area of rectangular field $(2x^2 + 11x + 14) \text{ m}^2$

Practice Set 35

1. Multiply.

(i) $16xy \times 18xy$

(ii) $23xy^2 \times 4yz^2$

(iii) $(12a + 17b) \times 4c$

(iv) $(4x + 5y) \times (9x + 7y)$

2. A rectangle is $(8x + 5)$ cm long and $(5x + 3)$ cm broad. Find its area.



Let's recall.

Equations in One Variable

Solve the following equations.

(1) $x + 7 = 4$

(2) $4p = 12$

(3) $m - 5 = 4$

(4) $\frac{t}{3} = 6$



Let's learn.

Example $2x + 2 = 8$
 $\therefore 2x + 2 - 2 = 8 - 2$
 $\therefore 2x = 6$
 $\therefore x = 3$

Example $3x - 5 = x - 17$
 $3x - 5 + 5 - x = x - 17 + 5 - x$
 $\therefore 2x = -12$
 $\therefore x = -6$

Example The length of a rectangle is 1 cm more than twice its breadth. If the perimeter of the rectangle is 50 cm, find its length.

Solution:

Let the breadth of the rectangle be x cm.

Then the length of the rectangle will be $(2x + 1)$ cm.

$2 \times \text{length} + 2 \times \text{breadth} = \text{perimeter of rectangle}$

$$2(2x + 1) + 2x = 50$$

$$\therefore 4x + 2 + 2x = 50$$

$$\therefore 6x + 2 = 50$$

$$\therefore 6x = 50 - 2 = 48$$

$$\therefore x = 8$$

Breadth of rectangle is 8 cm.

Length of the rectangle $= 2x + 1 = 2 \times 8 + 1$

\therefore Length of rectangle $= 17$ cm.

Example The sum of two consecutive natural numbers is 69. Find the numbers.

Solution:

Let one natural number be x .

The next natural number is $x + 1$

$$(x) + (x + 1) = 69$$

$$\therefore x + x + 1 = 69$$

$$\therefore 2x + 1 = 69$$

$$2x = 69 - 1$$

$$\therefore 2x = 68$$

$$\therefore x = 34$$

1st natural number $= 34$

2nd natural number $= 34 + 1$
 $= 35$

Remember :

From the solved examples above, we see that if a term is 'transposed' from one side to the other of the '=' sign in an equation, that term's sign must be changed.

Practice Set 36

- Simplify $(3x - 11y) - (17x + 13y)$ and choose the right answer.
(i) $7x - 12y$ (ii) $-14x - 54y$ (iii) $-3(5x + 4y)$ (iv) $-2(7x + 12y)$
- The product of $(23x^2y^3z)$ and $(-15x^3yz^2)$ is
(i) $-345x^5y^4z^3$ (ii) $345x^2y^3z^5$ (iii) $145x^3y^2z$ (iv) $170x^3y^2z^3$
- Solve the following equations.
(i) $4x + \frac{1}{2} = \frac{9}{2}$ (ii) $10 = 2y + 5$ (iii) $5m - 4 = 1$
(iv) $6x - 1 = 3x + 8$ (v) $2(x - 4) = 4x + 2$ (vi) $5(x + 1) = 74$
- Rakesh's age is less than Sania's age by 5 years. The sum of their ages is 27 years. How old are they?
- When planting a forest, the number of jambhul trees planted was greater than the number of ashoka trees by 60. If there are altogether 200 trees of these two types, how many jambhul trees were planted?
- Shubhangi has twice as many 20-rupee notes as she has 50-rupee notes. Altogether, she has 2700 rupees. How many 50-rupee notes does she have?
- * Virat made twice as many runs as Rohit. The total of their scores is 2 less than a double century. How many runs did each of them make?



Miscellaneous Problems : Set 1

1. Solve the following.

(i) $(-16) \times (-5)$

(ii) $(72) \div (-12)$

(iii) $(-24) \times (2)$

(iv) $125 \div 5$

(v) $(-104) \div (-13)$

(vi) $25 \times (-4)$
2. Find the prime factors of the following numbers and find their LCM and HCF.

(i) 75, 135

(ii) 114, 76

(iii) 153, 187

(iv) 32, 24, 48
- 3*. Simplify.

(i) $\frac{322}{391}$

(ii) $\frac{247}{209}$

(iii) $\frac{117}{156}$
4. Find the square root of the following numbers.

(i) 784

(ii) 225

(iii) 1296

(iv) 2025

(v) 256
5. There are four polling booths for a certain election. The numbers of men and women who cast their vote at each booth is given in the table below. Draw a joint bar graph for this data.

Polling Booths	Navodaya Vidyalaya	Vidyaniketan School	City High School	Eklavya School
Women	500	520	680	800
Men	440	640	760	600

6. Simplify the expressions.

(i) $45 \div 5 + 20 \times 4 - 12$

(ii) $(38 - 8) \times 2 \div 5 + 13$

(iii) $\frac{5}{3} + \frac{4}{7} \div \frac{32}{21}$

(iv) $3 \times \{ 4 [85 + 5 - (15 \div 3)] + 2 \}$
7. Solve.

(i) $\frac{5}{12} + \frac{7}{16}$

(ii) $3\frac{2}{5} - 2\frac{1}{4}$

(iii) $\frac{12}{5} \times \frac{(-10)}{3}$

(iv*) $4\frac{3}{8} \div \frac{25}{18}$
8. Construct $\triangle ABC$ such that $m\angle A = 55^\circ$, $m\angle B = 60^\circ$, and $l(AB) = 5.9$ cm.
9. Construct $\triangle XYZ$ such that, $l(XY) = 3.7$ cm, $l(YZ) = 7.7$ cm, $l(XZ) = 6.3$ cm.
10. Construct $\triangle PQR$ such that, $m\angle P = 80^\circ$, $m\angle Q = 70^\circ$, $l(QR) = 5.7$ cm.
11. Construct $\triangle EFG$ from the given measures. $l(FG) = 5$ cm, $m\angle EFG = 90^\circ$, $l(EG) = 7$ cm.
12. In $\triangle LMN$, $l(LM) = 6.2$ cm, $m\angle LMN = 60^\circ$, $l(MN) = 4$ cm. Construct $\triangle LMN$.
13. Find the measures of the complementary angles of the following angles.

(i) 35°

(ii) a°

(iii) 22°

(iv) $(40 - x)^\circ$
14. Find the measures of the supplements of the following angles.

(i) 111°

(ii) 47°

(iii) 180°

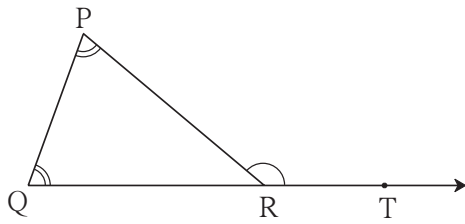
(iv) $(90 - x)^\circ$
15. Construct the following figures.

(i) A pair of adjacent angles

(ii) Two supplementary angles which are not adjacent angles.

(iii) A pair of adjacent complementary angles.

16.



In ΔPQR , the measures of $\angle P$ and $\angle Q$ are equal and $m\angle PRQ = 70^\circ$. Find the measures of the following angles.

(i) $m\angle PRT$ (ii) $m\angle P$ (iii) $m\angle Q$

17. Simplify.

(i) $5^4 \times 5^3$ (ii) $\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^9$ (iii) $\left(\frac{7}{2}\right)^8 \times \left(\frac{7}{2}\right)^{-6}$ (iv) $\left(\frac{4}{5}\right)^2 \div \left(\frac{5}{4}\right)$

18. Find the value.

(i) $17^{16} \div 17^{16}$ (ii) 10^{-3} (iii) $(2^3)^2$ (iv) $4^6 \times 4^{-4}$

19. Solve.

(i) $(6a - 5b - 8c) + (15b + 2a - 5c)$ (ii) $(3x + 2y)(7x - 8y)$
 (iii) $(7m - 5n) - (-4n - 11m)$ (iv) $(11m - 12n + 3p) - (9m + 7n - 8p)$

20. Solve the following equations.

(i) $4(x + 12) = 8$ (ii) $3y + 4 = 5y - 6$

Multiple Choice Questions

Choose the right answer from the options given after every question.

1. The three angle bisectors of a triangle are concurrent. Their point of concurrence is called the

(i) circumcentre (ii) apex (iii) incentre (iv) point of intersection.

2. $\left[\left(\frac{3}{7}\right)^{-3}\right]^4 = \dots\dots\dots$

(i) $\left(\frac{3}{7}\right)^{-7}$ (ii) $\left(\frac{3}{7}\right)^{-10}$ (iii) $\left(\frac{7}{3}\right)^{12}$ (iv) $\left(\frac{3}{7}\right)^{20}$

3. The simplest form of $5 \div \left(\frac{3}{2}\right) - \frac{1}{3}$ is

(i) 3 (ii) 5 (iii) 0 (iv) $\frac{1}{3}$

4. The solution of the equation $3x - \frac{1}{2} = \frac{5}{2} + x$ is

(i) $\frac{5}{3}$ (ii) $\frac{7}{2}$ (iii) 4 (iv) $\frac{3}{2}$

5*. Which of the following expressions has the value 37?

(i) $10 \times 3 + (5 + 2)$ (ii) $10 \times 4 + (5 - 3)$
 (iii) $8 \times 4 + 3$ (iv) $(9 \times 3) + 2$



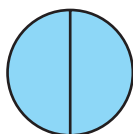


Let's discuss.

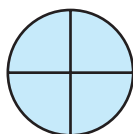
Direct proportion

In the previous class we have learnt how to compare two numbers and write them in the form of a ratio.

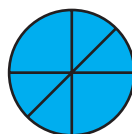
Example Look at the picture below. We see divisions of a circle made by its diameters.



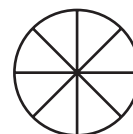
(A)



(B)



(C)



(D)

Do you see any relationship between the number of diameters and the number of divisions they give rise to?

In figure (A) **one** diameter makes parts of the circle.

In figure (B) **two** diameters make parts of the circle.

In figure (D) **four** diameters make parts of the circle.

$\frac{\text{No. of diameters}}{\text{No. of divisions}} = \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$. Here, the ratio of the number of diameters to the number of divisions remains constant.

Example The number of notebooks that the students of a Government School received is shown in the table below.

Children	15	12	10	5
Notebooks	90	72	60	30

$$\frac{\text{Number of children}}{\text{Number of notebooks}} = \frac{15}{90} = \frac{12}{72} = \frac{10}{60} = \frac{5}{30} = \frac{1}{6}$$

In other words, the ratio 1:6 remains the same or constant.

In the examples above, we see that when the number of diameters increases the number of divisions also increases. As the number of children decreases the number of notebooks also falls. The number of diameters and the number of divisions are in direct proportion as are the number of students and the number of notebooks.

Activity : ★ Think : Are the amount of petrol filled in a motorcycle and the distance travelled by it, in direct proportion?

★ Discuss : Can you give examples from science or everyday life, of quantities that vary in direct proportion?

Example If 10 pens cost 60 rupees, what is the cost of 13 such pens?

Solution : Let us suppose the cost of 13 pens is x rupees.

The number of pens and their cost vary in direct proportion. Let us express the ratios and obtain an equation.

$$\begin{aligned}\frac{10}{60} &= \frac{13}{x} \\ \therefore 10x &= 780 \text{ (multiplying both sides by } 60x) \\ \therefore x &= 78 \\ \text{Cost of 13 pens is } &\text{₹ } 78.\end{aligned}$$

Practice Set 37

1. If 7 kg onions cost 140 rupees, how much must we pay for 12 kg onions?
2. If 600 rupees buy 15 bunches of feed, how many will 1280 rupees buy?
3. For 9 cows, 13 kg 500 g of food supplement are required every day. In the same proportion, how much will be needed for 12 cows?
4. The cost of 12 quintals of soyabean is 36,000 rupees. How much will 8 quintals cost?
5. Two mobiles cost 16,000 rupees. How much money will be required to buy 13 such mobiles ?



Let's learn.

Inverse Proportion



Some volunteers have gathered to dig 90 pits for a tree plantation programme. One volunteer digs one pit in one day. If there are 15 volunteers, they will take $\frac{90}{15} = 6$ days to dig the pits.

10 volunteers will take $\frac{90}{10} = 9$ days.
Are the number of pits and the number of volunteers in direct proportion?

If the number of volunteers **decreases**, **more** days are required; and if the number of volunteers **increases**, **fewer** days are required for the job. However, the product of the number of days and number of volunteers remains constant. We say that these numbers are in **inverse** proportion.

• Suppose Sudha has to solve 48 problems in a problem set. If she solves 1 problem every day, she will need 48 days to complete the set. But, if she solves 8 problems every day, she will complete the set in $\frac{48}{8} = 6$ days and if she solves 12 problems a day, she will need $\frac{48}{12} = 4$ days. The number of problems solved in a day and the number of days needed are in inverse proportion. Their product is constant.

Thus, note that $8 \times 6 = 12 \times 4 = 48 \times 1$

Example Fifteen workers take 8 hours to build a wall. How many hours will 12 workers need to build the same wall?

Solution : As the number of workers increases, the number of hours decreases. The number of workers and number of hours are in inverse proportion. The product of the number of workers and the number of hours needed to build the wall is constant. Let us use the variable x to solve this problem.

Suppose, 12 workers take x hours. $12 \times x = 15 \times 8$

15 workers take 8 hours. $\therefore 12x = 120$

12 workers take x hours. $\therefore x = 10$

Thus, 12 workers will take 10 hours to build the wall.

Example A 40-page class magazine is to be written. If one student would require 80 days to write it, how many would 4 students require ?

Solution : If more students help to do the same task, fewer days will be required. That is, the number of students and number of days are in inverse proportion. Suppose 4 students need x days.

Students	Days
1	80
4	x

$$4x = 80 \times 1$$

$$x = \frac{80}{4}$$

$$x = 20$$

\therefore 4 students require 20 days.

Example Students of a certain school went for a picnic to a farm by bus. Here are some of their experiences. Say whether the quantities in each are in direct or in inverse proportion.

- Each student paid 60 rupees for the expenses.

As there were 45 students, rupees were collected.

Had there been 50 students, rupees would have been collected.

The number of students and money collected are in proportion.

- The sweets shop near the school gave 90 laddoos for the picnic.

If 45 students go for the picnic, each will get laddoos.

If 30 students go for the picnic, each will get laddoos.

The number of students and that of laddoos each one gets are in proportion.

- The farm is 120 km away from the school.

The bus went to the farm at a speed of 40 km per hour and took hours.

On the return trip, the speed was 60 km per hour. Therefore, it took hours.

The speed of the bus and the time it takes are in proportion.

- The farmer picked 180 *bors* from his trees.
He gave them equally to 45 students. Each student got *bors*.
Had there been 60 students, each would have got *bors*.

The number of students and the number of *bors* each one gets are in proportion

Practice Set 38

- Five workers take 12 days to weed a field. How many days would 6 workers take ? How many would 15 take ?
- Mohanrao took 10 days to finish a book, reading 40 pages every day. How many pages must he read in a day to finish it in 8 days?
- Mary cycles at 6 km per hour. How long will she take to reach her Aunt's house which is 12 km away? If she cycles at a speed of 4 km/hr, how long would she take ?
- The stock of grain in a government warehouse lasts 30 days for 4000 people. How many days will it last for 6000 people ?



Let's learn.

Partnership

When starting a business enterprise, money is required for an office, raw materials, etc. This amount is called the capital. Often, two or more people put in money for the capital. In other words, these people start a business by investing in the partnership. In a business partnership, all partners have a joint account in a bank. The profit made or the loss incurred is shared by the partners in proportion to the money each one has invested.

Example Jhelum and Atharva invested 2100 and 2800 rupees respectively and started a business. They made a profit of 3500 rupees. How should it be shared?

Solution : Let us find out the proportion of the investments.

$$2100:2800 = \frac{2100}{2800} = \frac{3}{4} = 3:4. \therefore \text{Proportion of investments is } 3:4.$$

The profit must also be shared in the same proportion.

Let Jhelum's profit be $3x$ and that of Atharva, $4x$. Then,

$$3x + 4x = 3500 \quad \text{as total profit is } 3500.$$

$$\therefore 7x = 3500 \quad \therefore x = 500$$

Jhelum's share = $3x = 1500$ rupees and Atharva's share = $4x = 2000$ rupees.

Example Chinmaya and Sam invested a total of 130000 rupees in a business in the proportion 3:2 respectively. What amount did each of them invest? If their total profit was 36000 rupees, what is the share of each?

Solution : The proportion of Chinmaya's and Sam's investment is 3:2.

The profit is shared in the same **proportion** as the investment, hence, proportion of profit is 3:2.

Let Chinmaya's investment be $3y$ and Sam's $2y$.

$$3y + 2y = \text{Total investment.}$$

$$\therefore 5y = 130000$$

$$\frac{5y}{5} = \frac{130000}{5} \dots\dots (\text{dividing by } 5)$$

$$\therefore y = 26000$$

$$\begin{aligned} \therefore \text{Chinmaya's investment} &= 3y \\ &= 3 \times 26000 \\ &= 78,000 \end{aligned}$$

$$\begin{aligned} \text{Sam's investment} &= 2y \\ &= 2 \times 26000 \\ &= 52000 \text{ rupees} \end{aligned}$$

Let Chinmaya's profit be $3x$ and Sam's $2x$.

$$3x + 2x = \text{Total Profit}$$

$$5x = 36000$$

$$\frac{5x}{5} = \frac{36000}{5} \dots\dots (\text{dividing by } 5)$$

$$\therefore x = 7200$$

$$\begin{aligned} \therefore \text{Chinmaya's profit} &= 3x \\ &= 3 \times 7200 \\ &= 21600 \end{aligned}$$

$$\begin{aligned} \text{Sam's profit} &= 2x \\ &= 2 \times 7200 \\ &= 14400 \text{ rupees} \end{aligned}$$

Example Abdul, Sejal and Soham each gave Sayali 30 rupees, 70 rupees and 50 rupees respectively. Sayali put in 150 rupees and bought paper, colours, etc. Together they made greeting cards and sold them all. If they made a total profit of 420 rupees, what was each one's share in the profit ?

Solution : The capital invested by all four was 300 rupees. Of this Sayali had invested 150 rupees, that is, half of the capital. The total profit was 420 rupees. So, Sayali's profit was half of that, i.e., 210 rupees. The remaining 210 was shared by Abdul, Sejal and Soham.

Abdul, Sejal and Soham's investment is 30, 70 and 50 rupees. The proportion is 30:70:50 i.e. 3:7:5. Their share of the profit is altogether 210 rupees.

Let their individual profit be $3k$, $7k$, $5k$. Then, $3k + 7k + 5k = 210$

$$\therefore 15k = 210$$

$$\therefore k = 14$$

$$\text{Abdul's profit} = 3k = 3 \times 14 = 42 \text{ rupees.}$$

$$\text{Sejal's profit} = 7k = 7 \times 14 = 98 \text{ rupees.}$$

$$\text{Soham's profit} = 5k = 5 \times 14 = 70 \text{ rupees.}$$

Example Saritaben, Ayesha and Meenakshi started a business by investing 2400, 5200 and 3400 rupees. They made a profit of 50%. How should they share it? If they reinvest all their profit by adding it to the capital, what will each one's share be in the following year?

Solution : Total capital = $2400 + 5200 + 3400 = 11000$ rupees.

Profit 50%

$$\therefore \text{Total profit} = \frac{11000 \times 50}{100} = 5500$$

Profit will be shared in the same proportion as the investment.

We simplify the ratio of two numbers by dividing by a common factor. The same can be done for 3 or more numbers.

$$\begin{aligned}\text{Proportion of shares} &= 2400 : 5200 : 3400 \\ &= 24 : 52 : 34 \quad (\text{dividing by } 100) \\ &= 12 : 26 : 17 \quad (\text{dividing by } 2)\end{aligned}$$

Assume that Saritaben's profit = 12p, Ayesha's profit = 26p, Meenakshi's profit = 17p.

$$\therefore 12p + 26p + 17p = 55p = 5500 \therefore p = \frac{5500}{55} = 100$$

\therefore Saritaben's profit = $12 \times 100 = 1200$, Ayesha's profit = $26 \times 100 = 2600$ and Meenakshi's profit = $17 \times 100 = 1700$.

If they add their profit to the capital, their further investments will be :

$$\text{Saritaben's capital} = 2400 + 1200 = ₹ 3600$$

$$\text{Ayesha's capital} = 5200 + 2600 = ₹ 7800$$

$$\text{Meenakshi's capital} = 3400 + 1700 = ₹ 5100$$



Let's discuss.

In the above example, if Saritaben, Meenakshi and Ayesha all add their profit to the capital, find out the proportions of their shares in the capital during the following year.

Practice Set 39

1. Suresh and Ramesh together invested 144000 rupees in the ratio 4:5 and bought a plot of land. After some years they sold it at a profit of 20%. What is the profit each of them got?
2. Virat and Samrat together invested 50000 and 120000 rupees to start a business. They suffered a loss of 20%. How much loss did each of them incur?
3. Shweta, Piyush and Nachiket together invested 80000 rupees and started a business of selling sheets and towels from Solapur. Shweta's share of the capital was 30000 rupees and Piyush's 12000. At the end of the year they had made a profit of 24%. What was Nachiket's investment and what was his share of the profit?
4. A and B shared a profit of 24500 rupees in the proportion 3:7. Each of them gave 2% of his share of the profit to the Soldiers' Welfare Fund. What was the actual amount given to the Fund by each of them?
- 5*. Jaya, Seema, Nikhil and Neelesh put in altogether 360000 rupees to form a partnership, with their investments being in the proportion 3:4:7:6. What was Jaya's actual share in the capital? They made a profit of 12%. How much profit did Nikhil make ?





Let's recall.

A bank is a government recognized institution that carries out transactions of money. Banks make it easier to plan the use of money, i.e., to do financial planning. We can either deposit cash money in a bank or withdraw cash from it. For that purpose, we must open an account in a bank. Bank accounts are of various kinds.



Let's learn.

Different Types of Accounts

* Current Account

A current account is mainly for traders and those dealing in money on a daily basis. An account holder can deposit or withdraw money any number of times in a day. The bank issues a passbook for this account and also a cheque book on demand. The bank does not pay any interest on the money in this type of account. Money can also be withdrawn or deposited by cheque.

* Savings Account

A person can deposit a minimum amount and open a savings account. In some banks, no minimum amount is required for opening an account. The bank pays some interest on the basis of the daily credit balance in the account. There are some restrictions on how often money can be withdrawn from this account. For this account too, the bank issues a passbook and, on demand, a cheque book.

* Recurring Deposit Account

The account holder can decide the amount to be deposited every month in the account. The bank gives an interest on the deposit which is more than that paid for the savings account. Such an account is a means of compulsory savings.

Often it is convenient to have a joint account for say, husband and wife or guardian and ward, etc. Besides, accounts of business partners, housing societies, trusts of voluntary agencies, etc. are required to be operated by more than one person.

* Fixed Deposit

A depositor deposits a certain amount for a fixed period in the bank. This deposit attracts a greater rate of interest than the savings account. However, these rates are different in different banks. Senior citizens get a slightly greater rate of interest than the usual.

ATM, credit and debit cards : An ATM (Automatic Teller Machine) card is used to withdraw cash without going to a bank. A credit card or debit card is used to carry out transactions without using cash. An account holder can get such a card on request to the bank.



Let's discuss.

Have you seen a bank passbook?

Observe the entries made in the page of a passbook shown below:

ओळ क्र. पंक्ति क्र. LINE NO.	तारीख दिनांक DATE	तपशील व्यौरा PARTICULARS	चेक क्रमांक चेक क्रमांक CHEQUE No.	रक्कम काढली निकाली गई रकम AMOUNT WITHDRAWN	रक्कम ठेवली जमा की गई रकम AMOUNT DEPOSITED	शिल्लक बाकी जमा BALANCE
1.	2.2.2016	cash			1500.00	7000.00
2.	8.2.2016	cheque	232069		5000.00	12000.00
3.	12.2.2016	cheque	243965	3000.00		9000.00
4.	15.2.2016	self		1500.00		7500.00
5.	26.2.2016	interest			135.00	7635.00

- On 2.2.16 the amount deposited was rupees and the balance rupees.
- On 12.2.16 rupees were withdrawn by cheque no. 243965. The balance was rupees.
- On 26.2.2016 the bank paid an interest of rupees.

A passbook is issued for a savings account and a recurring deposit account. Amounts deposited, withdrawn and the balance are recorded in it with their dates.

Activity: Ask an adult in your house to show you a passbook and explain the entries made in it.



Let's recall.

Suvidya borrowed a sum of 30000 rupees at 8 p.c.p.a. interest for a year from her bank to buy a computer. At the end of the period, she had to pay back an amount of 2400 rupees over and above what she had borrowed.

- Based on this information fill in the boxes below.

Principal = ₹ , Rate of interest = %, Interest = ₹ , Time = years.

The total amount returned to the bank = $30,000 + 2,400 =$



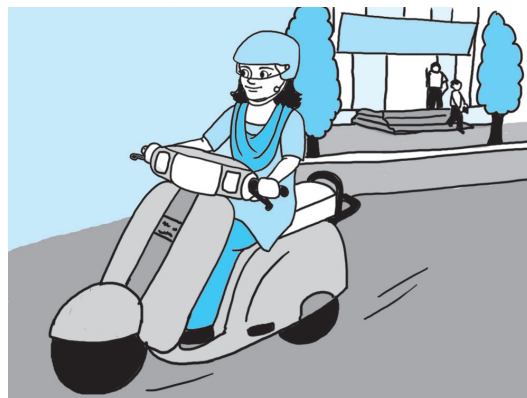
Let's learn.

We added the capital and the interest accrued on it to find out the amount that Suvidya returned to the bank. Thus,

$$\text{Principal} + \text{Interest} = \text{Amount}$$

Example Neha took a loan of 50000 rupees at 12 p.c.p.a. to buy a two wheeler. What amount will she return to the bank at the end of one year?

Solution : The amount, that is, the total money owed to the bank at the end of the time, is to be calculated here. The principal is 50000 rupees. At 12 p.c.p.a., the interest on 100 rupees for one year is 12 rupees. We shall write the ratio of interest to capital in two ways to obtain an equation.



On 50000 rupees let the interest be x rupees.

On 100 rupees the interest is 12 rupees.

$$\frac{x}{50000} = \frac{12}{100}$$

$$\frac{x}{50000} \times 50,000 = \frac{12}{100} \times 50,000 \quad (\text{Multiplying both sides by } 50000)$$

$$x = 6000$$

$$\begin{aligned} \text{Amount (to be returned to the bank)} &= \text{principal} + \text{interest} \\ &= 50,000 + 6,000 \end{aligned}$$

$$\therefore \text{Amount to be returned to the bank} = ₹ 56,000$$

Example Aakash deposited 25000 rupees in a bank at a rate of 8 p.c.p.a. for 3 years. How much interest does he get every year? How much, altogether?

Solution : Here, the principal is 25000 rupees, time is 3 years and rate of interest is 8 on 100 rupees. The interest on 100 rupees is 8 rupees. Let us suppose the interest on 25000 rupees for 1 year is x . Let us find the ratio of interest to principal. Then,

$$\frac{x}{25000} = \frac{8}{100}$$

$$\therefore \frac{x}{25000} \times 25000 = \frac{8}{100} \times 25000 \quad (\text{Multiplying both sides by } 25000)$$

$$\therefore x = 2000$$

Aakash got 2000 rupees interest for one year.

For three years he got $= 2000 \times 3 = 6000$ rupees interest.



Let's learn.

Let's learn the formula used to solve problems based on simple interest.

The principal is the same every year and the rate of interest too remains the same. The interest calculated in this way is called simple interest. Let us calculate the total interest when the principal P is deposited for T years at R rate of interest. Suppose the interest for one year is I.

The ratio of interest to principal for one year:

$$\frac{I}{P} = \frac{R}{100} \quad \therefore I = \frac{P \times R}{100}$$

$$\text{Interest for T years} = I \times T = \frac{P \times R \times T}{100}$$

$$\therefore \text{Total interest I} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

Solving the same example using the formula:

$$\text{Principal} = P = 25000, R = 8, T = 3$$

$$\text{Total interest} = \frac{P \times R \times T}{100}$$

$$= \frac{25000 \times 8 \times 3}{100}$$

$$= 6000$$

\therefore Total interest is 6000 rupees.



Now I know!

$$\text{Total interest I} = \frac{P \times T \times R}{100} \text{ where } P = \text{principal, } T = \text{time in years, } R = \text{rate of interest}$$

Example Sandeepbhau borrowed 120000 rupees from a bank for 4 years at the rate of $8\frac{1}{2}$ p.c.p.a. for his son's education. What is the total amount he returned to the bank at the end of that period ?

Solution: Principal = 120000, P = 120000, R = 8.5, T = 4

$$\therefore \text{Total interest} = \frac{P \times R \times T}{100} = \frac{120000 \times 8.5 \times 4}{100}$$

$$= \frac{120000 \times 85 \times 4}{100 \times 10}$$

$$= 120 \times 85 \times 4$$

$$= 40800$$

The total amount returned to the bank = 120000 + 40800 = 160800 rupees.

Practice Set 40

1. If Rihanna deposits 1500 rupees in the school fund at 9 p.c.p.a for 2 years, what is the total amount she will get?
2. Jethalal took a housing loan of 2,50,000 rupees from a bank at 10 p.c.p.a. for 5 years. What is the yearly interest he must pay and the total amount he returns to the bank?
- 3*. Shrikant deposited 85,000 rupees for $2\frac{1}{2}$ years at 7 p.c.p.a. in a savings bank account. What is the total interest he received at the end of the period?
4. At a certain rate of interest, the interest after 4 years on 5000 rupees principal is 1200 rupees. What would be the interest on 15000 rupees at the same rate of interest for the same period?
5. If Pankaj deposits 1,50,000 rupees in a bank at 10 p.c.p.a. for two years, what is the total amount he will get from the bank?



Let's learn.

When three of the four quantities, principal, time, rate and amount are given, to find the fourth: In the formula, we place any letter in place of the unknown quantity and solve the equation thus obtained to find the answer.

Example Principal = 25000 rupees, Amount = 31,000 rupees, Time = 4 years, what is the rate of interest?

Here, Amount – Principal = Total interest

$$31000 - 25000 = 6000$$

Principal = 25000 rupees, time = 4 years, interest = 6000 rupees

Let us now use the formula.

$$\text{Simple interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

$$\text{Then, } 6000 = \frac{25000 \times R \times 4}{100}$$

where R = rate of interest

$$R = \frac{6000 \times 100}{25000 \times 4}$$

$$\therefore R = 6$$

\therefore Rate of interest is 6 p.c.p.a

Example Unmesh borrowed some money for 5 years at simple interest. The rate of interest is 9 p.c.p.a. If he returned 17400 rupees altogether at the end of 5 years, how much had he borrowed?

$$\text{Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

This formula cannot be used directly to solve the problem because we do not

know either interest or principal. However, the interest on a principal of 100 rupees for 5 years is 45 rupees. Hence, the amount is $100 + 45 = 145$ rupees. Now we can express the ratio of principal and amount in two ways and obtain an equation.

$$\text{If Unmesh's principal is P then } \frac{P}{17400} = \frac{100}{145}$$

$$\therefore P = \frac{100 \times 17400}{145} = 12000$$

\therefore The principal that Unmesh borrowed was 12000 rupees.



Let's discuss.

- Can we solve the problem by using the formula to obtain a different kind of equation?

Practice Set 41

- If the interest on 1700 rupees is 340 rupees for 2 years the rate of interest must be
(i) 12 % (ii) 15 % (iii) 4 % (iv) 10 %
- If the interest on 3000 rupees is 600 rupees at a certain rate for a certain number of years, what would the interest be on 1500 rupees under the same conditions ?
(i) 300 rupees (ii) 1000 rupees (iii) 700 rupees (iv) 500 rupees
- Javed deposited 12000 rupees at 9 p.c.p.a. in a bank for some years, and withdrew his interest every year. At the end of the period, he had received altogether 17,400 rupees. For how many years had he deposited his money ?
- * Lataben borrowed some money from a bank at a rate of 10 p.c.p.a. interest for $2\frac{1}{2}$ years to start a cottage industry. If she paid 10250 rupees as total interest, how much money had she borrowed ?
- Fill in the blanks in the table.

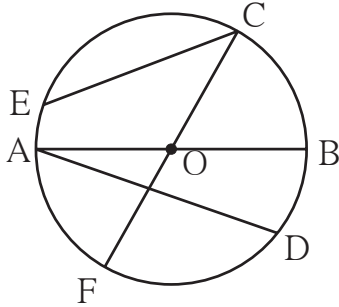
	Principal	Rate of interest (p.c.p.a.)	Time	Interest	Amount
(i)	4200	7%	3 years
(ii)	6%	4 years	1200
(iii)	8000	5%	800
(iv)	5%	6000	18000
(v)	$2\frac{1}{2}$ %	5 years	2400

Activity : ★ Visit different banks and find out the rates of the interest they give for different types of accounts.

- ★ With the help of your teachers, start a Savings Bank in your school and open an account in it to save up some money.



Let's recall.

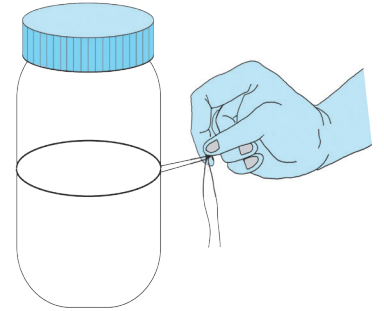


- Identify the radii, chords and diameters in the circle alongside and write their names in the table below.

Radii				
Chords				
Diameters				

Circumference of a Circle

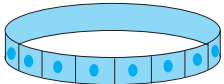

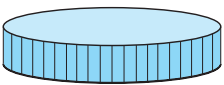
- Activity I** Place a cylindrical bottle on a paper and trace the outline of its base. Use a thread to measure the circumference of the circle.
- Activity II** Measure the circumference of a bangle with the help of a thread.
- Activity III** Measure the circumference of any cylindrical object using a thread.



Let's learn.

Relationship between Circumference and Diameter

- Activity** Measure the circumference and diameter of the objects given below and enter the ratio of the circumference to its diameter in the table.

Sr. No.	Object	Circumference	Diameter	Ratio $\frac{C}{D}$
1.	 Bangle	19 cm	6 cm	$\frac{19}{6} = 3.16$
2.	 Circular dish
3.	 Lid of a jar

Examine the ratio of the circumference to the diameter. What do we see?

The ratio of the circumference of any circle to its diameter is a little over 3 and remains constant. This constant is represented by the Greek letter π . Great mathematicians have proved through hard work that this number is not a rational number. In practice, the value of π is taken to be $\frac{22}{7}$ or 3.14. If the value of π has not been given in a problem, it is taken to be $\frac{22}{7}$.

If radius is ' r ', diameter ' d ' and circumference ' c ', $\frac{\text{circumference}(c)}{\text{diameter}(d)} = \pi$ $c = \pi d$

But $d = 2r$ $\therefore c = \pi \times 2r$ or $c = 2\pi r$

Example The diameter of a circle is 14 cm. Find its circumference.

Solution: Diameter : $d = 14$ cm
Circumference = πd
 $c = \frac{22}{7} \times 14$
Circumference of the circle = 44 cm

Example The circumference of a circle is 198 cm. Find its radius and diameter.

Solution: Circumference $c = 2\pi r$
 $198 = 2 \times \frac{22}{7} \times r$
 $r = 198 \times \frac{1}{2} \times \frac{7}{22}$
Radius = 31.5 cm
 \therefore Diameter = $2 \times 31.5 = 63$ cm.

Example The radius of a circle is 35 cm. Find its circumference.

Solution: Radius of the circle $r = 35$ cm
Circumference = $2\pi r$
 $c = 2 \times \frac{22}{7} \times 35$
Circumference of the circle = 220 cm.

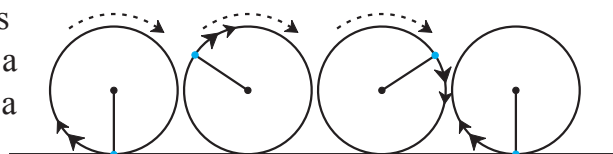
Example The circumference of a circle is 62.80 cm. Taking $\pi = 3.14$, find its diameter.

Solution: Circumference $c = \pi d$
 $62.80 = 3.14 \times d$
 $\frac{62.80}{3.14} = d$
 $20 = d$
 \therefore Diameter = 20 cm

Example The radius of a circular plot is 7.7 metres. How much will it cost to fence the plot with 3 rounds of wire at the rate of 50 rupees per metre?

Solution : Circumference of circular plot = $2\pi r = 2 \times \frac{22}{7} \times 7.7 = 48.4$
Length of wire required for one round of fencing = 48.4 m.
Cost of one round of fence = length of wire \times cost per metre.
 $= 48.4 \times 50$
 $= 2420$ rupees.
Cost of 3 rounds of fencing = $3 \times 2420 = 7260$ rupees

Example The radius of the wheel of a bus is 0.7 m. How many rotations will a wheel complete while travelling a distance of 22 km?



Solution: Circumference of circle = πd

$$= \frac{22}{7} \times 0.7$$

$$= 2.2 \text{ m}$$

When finding the ratio of like terms, their units must be the same.
22 km = $22 \times 1000 = 22000 \text{ m}$.

When the wheel completes one rotation it crosses a distance of 2.2 m.,
(1 rotation = 1 circumference)

$$\text{Total number of rotations} = \frac{\text{distance}}{\text{circumference}} = \frac{22000}{2.2} = \frac{220000}{22} = 10000$$

A wheel completes 10000 rotations to cover the distance of 22 km.

Practice Set 42

1. Complete the table below.

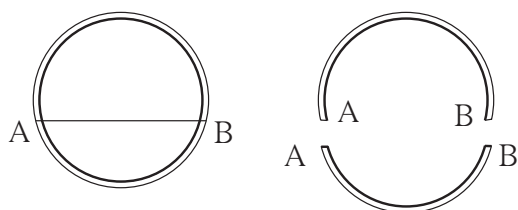
Sr. No.	Radius (r)	Diameter (d)	Circumference (c)
(i)	7 cm
(ii)	28 cm
(iii)	616 cm
(iv)	72.6 cm

- If the circumference of a circle is 176 cm, find its radius.
- The radius of a circular garden is 56 m. What would it cost to put a 4-round fence around this garden at a rate of 40 rupees per metre ?
- The wheel of a bullock cart has a diameter of 1.4m. How many rotations will the wheel complete as the cart travels 1.1 km ?



Let's recall.

Arc of the Circle



A plastic bangle is shown alongside. Suppose it breaks at points A and B. What is each of these pieces called as a part of a circle?

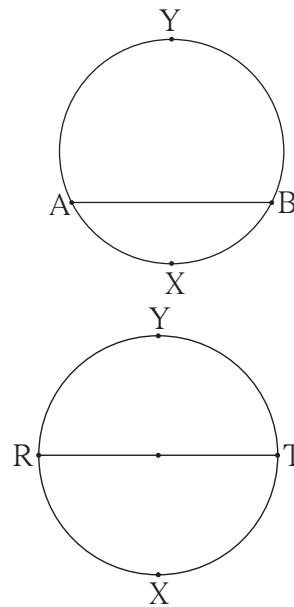


Let's learn.

The chord AB divides the circle alongside into two parts. Of these, the arc AXB is smaller and is called a **minor arc**. Arc AYB is bigger and is called the **major arc**. Minor arc AXB is also expressed as arc AB.

If two arcs of a circle have common end points and the arcs make one complete circle, the arcs are said to be corresponding arcs. Here, arc AYB and arc AXB are mutually corresponding arcs.

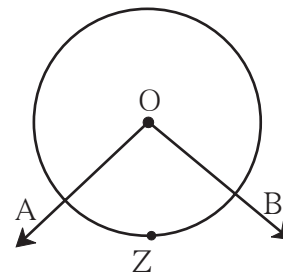
In the figure alongside, chord RT is a diameter of the circle. The diameter gives rise to two equal arcs. They are called **semicircular arcs**.



★ Central Angle and the Measure of an Arc

In the figure, 'O' is the vertex of the $\angle AOB$. An angle whose vertex is the centre of the circle is called a **central angle**.

The $\angle AOB$ in the figure is the central angle corresponding to arc AZB. **The measure of the angle subtended at the centre by an arc is taken to be the measure of the arc.**

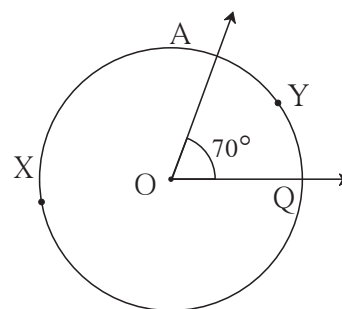


★ The measure of a minor arc

In the figure alongside, the measure of $\angle AOQ = 70^\circ$.

\therefore Measure of the minor arc AYQ is 70°

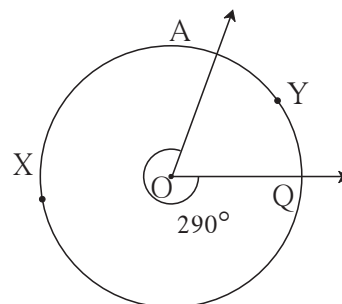
It is written as $m(\text{arc AYQ}) = 70^\circ$



★ The measure of a major arc

Measure of a major arc = 360° – measure of the corresponding minor arc

\therefore Measure of major arc AXQ in the figure = $360^\circ - 70^\circ = 290^\circ$

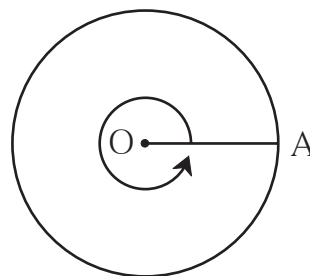


★ **The measure of a circle**

When the radius OA of a circle turns anti-clockwise, as shown in the figure alongside, through a complete angle, it turns through an angle that measures 360° . Its end point A completes one circle.

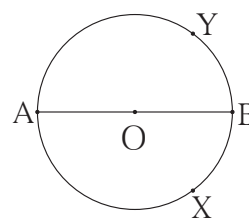
\therefore The angle subtended at the centre by the circle is 360° .

\therefore The measure of the complete circle is 360° .



★ **Measure of a semicircular arc**

Now, look at the figure and determine the measures of the semicircular arcs AXB and AYB.



Now I know!

- The measure of a minor arc is equal to its corresponding central angle.
- The measure of a major arc = $360^\circ -$ measure of corresponding minor arc.
- The measure of a semicircular arc = 180°

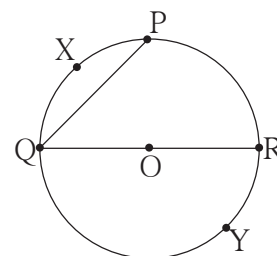
Practice Set 43

1. Choose the correct option.

If arc AXB and arc AYB are corresponding arcs and $m(\text{arc AXB}) = 120^\circ$ then $m(\text{arc AYB}) =$.

- (i) 140° (ii) 60° (iii) 240° (iv) 160°

2. Some arcs are shown in the circle with centre 'O'. Write the names of the minor arcs, major arcs and semicircular arcs from among them.



3. In a circle with centre O, the measure of a minor arc is 110° . What is the measure of the major arc PYQ?



ICT Tools or Links

Using the MOVE option of the Geogebra software, observe the relationship between a central angle and its corresponding arc, for different measures of the arc.





Let's recall.

Perimeter

The sum of the lengths of all sides of a closed figure is the perimeter of that figure.
Perimeter of a polygon = sum of lengths of all sides.

∴ Perimeter of a square = $4 \times \text{side}$

Perimeter of square of side $a = 4a$

Perimeter of a rectangle = $2 \text{ length} + 2 \text{ breadth}$

Perimeter of a rectangle with length l and breadth b is $2l + 2b$

Example The perimeter of a rectangle is 64 cm. If its length is 17 cm, what is its breadth?

Solution : Let its breadth be x cm.

$$2 \text{ length} + 2 \text{ breadth} = \text{perimeter}$$

$$2 (\text{length} + \text{breadth}) = 64$$

$$2 (17 + x) = 64$$

$$\frac{2(17+x)}{2} = \frac{64}{2}$$

$$17 + x = 32$$

$$x = 15$$

The breadth of the rectangle is 15 cm.

Example The perimeter of a rectangle of length 28 cm and breadth 20 cm is equal to the perimeter of a square. What is the length of the side of that square?

Solution : Perimeter of rectangle

$$= 2 (\text{length} + \text{breadth})$$

$$= 2 (28 + 20)$$

$$= 96$$

If the side of that square is a then $4a = 96$

$$\text{Perimeter of square} = 96$$

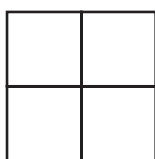
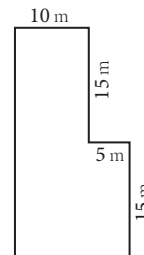
$$4a = 96$$

$$\therefore a = \frac{96}{4} = 24$$

Side of the square is 24 cm.

Practice Set 44

- If the length and breadth of a rectangle are doubled, how many times the perimeter of the old rectangle will that of the new rectangle be?
- If the side of a square is tripled, how many times the perimeter of the first square will that of the new square be?
- Given alongside is the diagram of a playground. It shows the length of its sides. Find the perimeter of the playground.



- As shown in the figure, four napkins all of the same size were made from a square piece of cloth of length 1 m. What length of lace will be required to trim all four sides of all the napkins ?



Let's recall.

Area

- Area of square = side \times side = (side)²
- Area of rectangle = length \times breadth = $l \times b$

Area is measured in square metres, square cm, square km, etc.

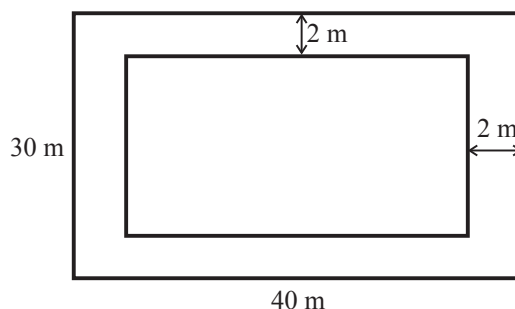
Activity I

Measure the length and breadth of the courts laid out for games such as kho-kho, kabaddi, tennis, badminton, etc. Find out their perimeters and areas.

Activity II

A wall in Aniruddha's house is to be painted. The wall is 7 m long and 5 m high. If the painter charges 120 rupees per square metre, how much will he have to be paid?

Example A rectangular garden is 40 m long and 30 m wide. A two-metre wide path is to be paved inside the garden along its boundary, using tiles 25 cm \times 20 cm in size. How many such tiles will be required ?



Let us find the area to be paved.

$$\text{Area of garden} = 40 \times 30 = 1200 \text{ sqm}$$

$$\text{Area not to be paved} = 36 \times 26 = 936 \text{ sqm}$$

$$\therefore \text{Area to be paved} = 1200 - 936 = 264 \text{ sqm}$$

$$\text{Area of each tile} = \frac{25}{100} \times \frac{20}{100} = \frac{1}{20} \text{ sqm}$$

$$\text{Area of one tile is } \frac{1}{20} \text{ sqm.}$$

Hence, let us find the number of tiles required to tile 264 sqm.

$$\text{Number of tiles} = \frac{\text{Total area}}{\text{Area of one paver}}$$

$$= 264 \div \frac{1}{20}$$

$$= 264 \times 20 = 5280$$

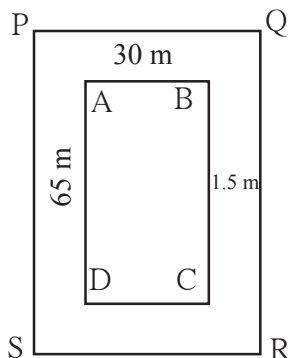
Therefore, 5280 tiles will be required.

$$100 \text{ cm} = 1 \text{ m}$$

$$25 \text{ cm} = \frac{25}{100} \text{ m}$$

Example A rectangular playground is 65 m long and 30 m wide. A pathway of 1.5 m width goes all around the ground, outside it. Find the area of the pathway.

Solution: The playground is rectangular.



□ ABCD is the playground. Around it is a pathway 1.5 m wide.

Around □ ABCD we get the rectangle □ PQRS

Length of new rectangle PQRS = $65 + 1.5 + 1.5 = 68$ m

Breadth of new rectangle PQRS = $30 + 1.5 + 1.5 = 33$ m

Area of path = Area of rectangle PQRS – Area of rectangle ABCD

$$= 68 \times 33 - 65 \times 30 = \boxed{} - \boxed{} = \boxed{} \text{ sqm.}$$



Let's discuss.

- Is there another way to find the area of the pathway in the problem above?

Example The length and the width of a mobile phone are 13 cm and 7 cm respectively. It has a screen PQRS as shown in the figure.

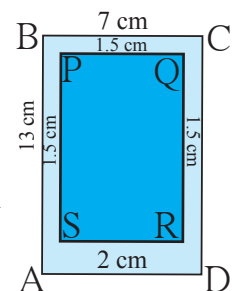
What is the area of the screen?

Solution: ABCD is the rectangle formed by the edges of the mobile. PQRS is the rectangle formed by leaving a 1.5 cm wide edge alongside AB, BC, and DC, and a 2 cm edge alongside DA.

Length of rectangle PQRS = $\boxed{}$ cm

Breadth of rectangle PQRS = $\boxed{}$ cm

Area of screen = Area of rectangle PQRS = $\dots \times \dots = \boxed{}$ sqm



Activity

Take mobile handsets of different sizes and find the area of their screens.

Practice Set 45

1. If the side of a square is 12 cm, find its area.
2. If the length of a rectangle is 15 cm and breadth is 5 cm, find its area.
3. The area of a rectangle is 102 sqcm. If its length is 17 cm, what is its perimeter ?
- 4*. If the side of a square is tripled, how many times will its area be as compared to the area of the original square ?



Let's learn.

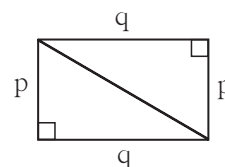
Area of a Right-angled Triangle

Activity Cut out two right-angled triangles having the same measures. Join them as shown in the figure. See how they form a rectangle. The sides of length p and q that form the right angles of the triangles are also the ones that form the sides of the rectangle. From the figure we see that

Area of a rectangle = $2 \times$ area of right-angled triangle

$\therefore 2 \times$ area of right-angled triangle = $p \times q$

$$\text{Area of right-angled triangle} = \frac{p \times q}{2}$$



Now I know!

Area of a right-angled triangle = $\frac{1}{2} \times$ product of sides forming the right angle

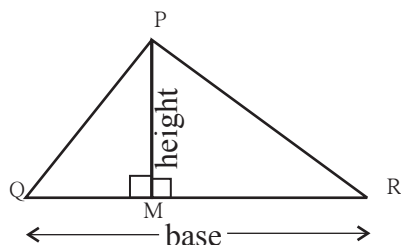
If one of the sides of a right-angled triangle forming the right angle is taken to be the base, the other becomes the height of the triangle.

Thus, the area of a right-angled triangle = $\frac{1}{2}$ base \times height

If $\triangle ABC$ is any triangle, then any side can be taken as the base. Then the measure of the perpendicular on the base from the apex opposite to it, is the height of the triangle.

Take any $\triangle PQR$ and take QR as the base. PM is the perpendicular from P on QR .

Figure 1 : Point M is in seg QR .

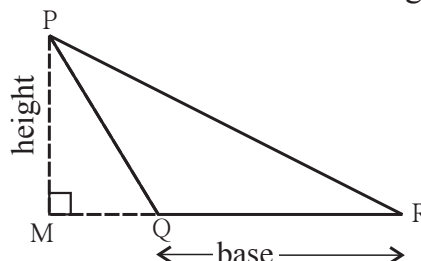


$\triangle PMR$ and $\triangle PMQ$ are right-angled triangles.

$$\begin{aligned} A(\triangle PQR) &= A(\triangle PMQ) + A(\triangle PMR) \\ &= \frac{1}{2} \times l(QM) \times l(PM) + \frac{1}{2} \times l(MR) \times l(PM) \\ &= \frac{1}{2} [l(QM) + l(MR)] \times l(PM) \\ &= \frac{1}{2} l(QR) \times l(PM) \\ &= \frac{1}{2} \times \text{base} \times \text{height} \end{aligned}$$

$$A(\triangle PQR) = \frac{1}{2} \times \text{base} \times \text{height}$$

Figure 2 : Point M is outside seg QR .



$\triangle PMR$ and $\triangle PMQ$ are right-angled triangles.

$$\begin{aligned} A(\triangle PQR) &= A(\triangle PMR) - A(\triangle PMQ) \\ &= \frac{1}{2} \times l(MR) \times l(PM) - \frac{1}{2} \times l(MQ) \times l(PM) \\ &= \frac{1}{2} [l(MR) - l(MQ)] \times l(PM) \\ &= \frac{1}{2} \times l(QR) \times l(PM) \\ &= \frac{1}{2} \times \text{base} \times \text{height} \end{aligned}$$

$$A(\triangle PQR) = \frac{1}{2} \times \text{base} \times \text{height}$$



Now I know!

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

Example If the sides that form the right angle of a triangle are 3.5 cm and 4.2 cm long, find the area of the triangle.

Solution: Area of a right-angled triangle

$$\begin{aligned} &= \frac{1}{2} \times \text{product of sides forming right angle} \\ &= \frac{1}{2} \times 3.5 \times 4.2 \\ &= 7.35 \text{ sqcm} \end{aligned}$$

Example If the base of a triangle is 5.6 cm and height is 4.5 cm, what is its area?

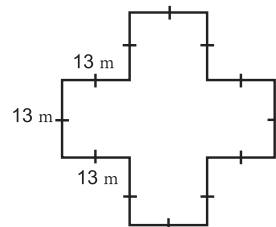
Solution: Area of a triangle

$$\begin{aligned} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 5.6 \times 4.5 \\ &= 12.6 \text{ sqcm} \end{aligned}$$

(Note that sqcm is also written as cm^2).

Pratice set 46

1. A page of a calendar is 45 cm long and 26 cm wide. What is its area ?
2. What is the area of a triangle with base 4.8 cm and height 3.6 cm ?
3. What is the value of a rectangular plot of land 75.5 m long and 30.5 m broad at the rate of 1000 rupees per square metre?
4. A rectangular hall is 12 m long and 6 m broad. Its flooring is to be made of square tiles of side 30 cm. How many tiles will fit in the entire hall? How many would be required if tiles of side 15 cm were used?
5. Find the perimeter and area of a garden with measures as shown in the figure alongside.

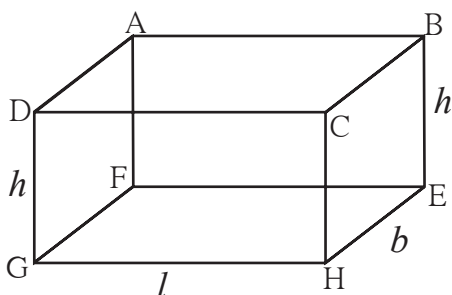


Let's learn.

Surface Area

The surface area of any three-dimensional object is the sum of the areas of all its faces.

* Surface Area of a Cuboid



- A cuboid has six faces.
- Each face is a rectangle.
- Opposite faces have the same area.
- Each edge is perpendicular to the two other edges it meets.
- Let l be the length of the horizontal face of the cuboid and b be the breadth. Let h be the height of its vertical sides.

Area of rectangle ABCD = Area of rectangle GHEF = $l \times b$

Area of rectangle ADGF = Area of rectangle BCHE = $b \times h$

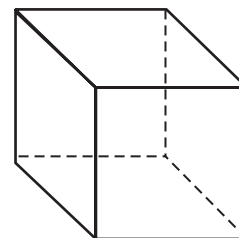
Area of rectangle CHGD = Area of rectangle ABEF = $l \times h$

Total surface area of cuboid = Sum of area of all rectangles

$$\begin{aligned}\text{Total surface area of cuboid} &= 2 (\text{length} \times \text{breadth} + \text{breadth} \times \text{height} + \text{length} \times \text{height}) \\ &= 2 (l \times b + b \times h + l \times h) = 2 (lb + bh + lh)\end{aligned}$$

★ Surface Area of a Cube

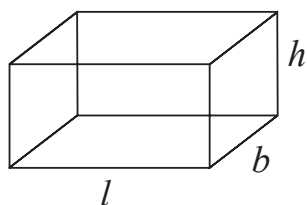
- A cube has 6 faces.
- Each face is a square.
- Area of all faces is equal.
- Let the side of each square be l .
- Area of one face of the cube = Area of square
- Total surface area of the cube = Sum of areas of 6 squares.
 $= 6 \times \text{side}^2$
 $= 6 \times l^2$



Example How much sheet metal is required to make a closed rectangular box of length 1.5 m, breadth 1.2 m and height 1.3 m ?

Solution : length of box = $l = 1.5$ m, breadth = $b = 1.2$ m, height = $h = 1.3$ m.

$$\begin{aligned}\text{Surface area of box} &= 2 (l \times b + b \times h + l \times h) \\ &= 2 (1.5 \times 1.2 + 1.2 \times 1.3 + 1.5 \times 1.3) \\ &= 2 (1.80 + 1.56 + 1.95) \\ &= 2 (5.31) \\ &= 10.62 \text{ sqm}\end{aligned}$$



10.62 sqm of sheet metal will be needed to make the box.

Example One side of a cubic box is 0.4 m. How much will it cost to paint the outer surface of the box at the rate of 50 rupees per sqm?

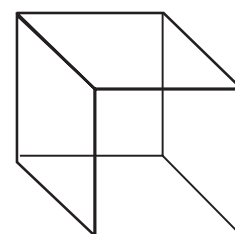
Solution : side = $l = 0.4$ m.

$$\begin{aligned}\text{Total surface area of cube} &= 6 \times (l)^2 \\ &= 6 \times (0.4)^2 \\ &= 6 \times 0.16 = 0.96 \text{ sqm}\end{aligned}$$

Cost of painting 1 sqm is 50 rupees.

$$\begin{aligned}\therefore \text{Cost of painting } 0.96 \text{ sqm will be} &= 0.96 \times 50 \\ &= 48 \text{ rupees}\end{aligned}$$

It will cost 48 rupees to paint the outer surface of the box.



Practice Set 47

- Find the total surface area of cubes having the following sides.
(i) 3 cm (ii) 5 cm (iii) 7.2 m (iv) 6.8 m (v) 5.5 m
- Find the total surface area of the cuboids of length, breadth and height as given below:
(i) 12 cm, 10 cm, 5 cm (ii) 5 cm, 3.5 cm, 1.4 cm
(iii) 2.5 cm, 2 m, 2.4 m (iv) 8 m, 5 m, 3.5 m
- A matchbox is 4 cm long, 2.5 cm broad and 1.5 cm in height. Its outer sides are to be covered exactly with craft paper. How much paper will be required to do so ?
- An open box of length 1.5 m, breadth 1 m, and height 1 m is to be made for use on a trolley for carrying garden waste. How much sheet metal will be required to make this box? The inside and outside surface of the box is to be painted with rust proof paint. At a rate of 150 rupees per sqm, how much will it cost to paint the box?

Maths is fun !

There are some three-digit numbers which can be divided by the product of their digits without leaving a remainder.

Example (i) Take the number 175, $1 \times 7 \times 5 = 35$, $\frac{175}{35} = 5$

(ii) Take the number 816, $8 \times 1 \times 6 = 48$, $\frac{816}{48} = 17$

(iii) Take the number 612, $6 \times 1 \times 2 = 12$, $\frac{612}{12} = 51$

The numbers 135, 312, 672 are some more numbers like these.

Can you find other such numbers?



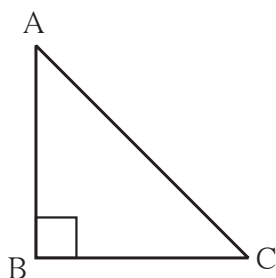


Let's recall.

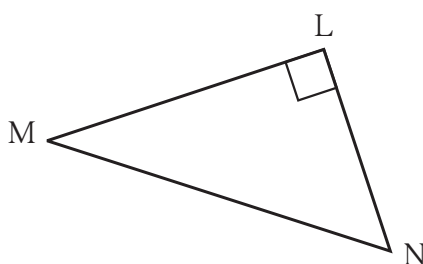
Right-angled Triangle

We know that a triangle with one right angle is called a right-angled triangle and the side opposite to the right angle is called the hypotenuse.

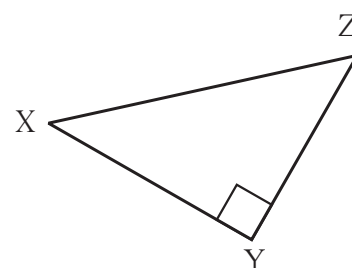
- Write the name of the hypotenuse of each of the right-angled triangles shown below.



The hypotenuse of $\triangle ABC$



The hypotenuse of $\triangle LMN$



The hypotenuse of $\triangle XYZ$

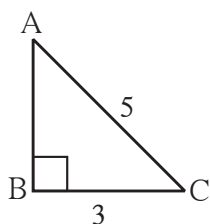
Pythagoras' Theorem

Pythagoras was a great Greek mathematician of the 6th century BCE. He made important contributions to mathematics. His method of teaching mathematics was very popular. He trained several mathematicians.

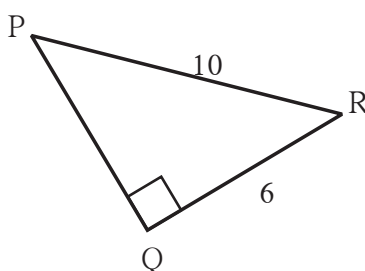
People of many countries had long known of a certain principle related to the right-angled triangle. It is also given in the book called Shulvasutra, of ancient India. As Pythagoras was the first to prove the theorem, it is named after him. This theorem of Pythagoras states that **in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.**

Activity : Draw right-angled triangles given the hypotenuse and one side as shown in the rough figures below. Measure the third side. Verify Pythagoras' theorem.

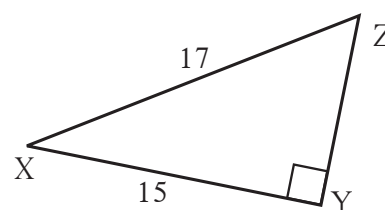
(i)



(ii)

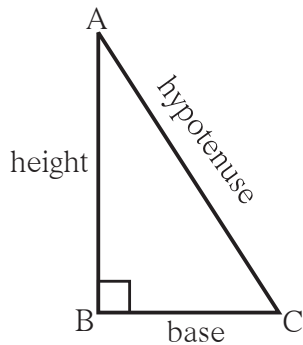


(iii)





Let's learn.



With reference to the figure alongside, the theorem of Pythagoras can be written as follows:

In $\triangle ABC$, if $\angle B$ is a right angle, then
 $[l(AC)]^2 = [l(AB)]^2 + [l(BC)]^2$

Generally, in a right-angled triangle, one of the sides forming the right angle is taken as the base and the other as the height.

Then, the theorem can be stated as
(hypotenuse)² = (base)² + (height)².

Follow the steps given below to **verify Pythagoras' theorem**.

Activity : From a cardsheet, cut out eight identical right-angled triangles. Let us say the length of the hypotenuse of these triangles is 'a' units, and sides forming the right angle are 'b' and 'c' units. Note that the area of this triangle is $\frac{bc}{2}$. Next, on another cardsheet, use a pencil to draw two squares ABCD and PQRS each of side (b + c) units. Now, place 4 of the triangle cut-outs in the square ABCD and the remaining 4 in the square PQRS as shown in the figures below. Mark by lines drawn across them, the parts of the squares covered by the triangles.

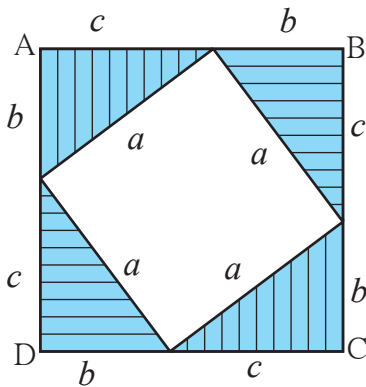
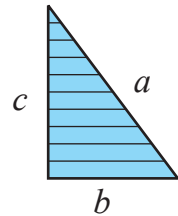


Figure (i)

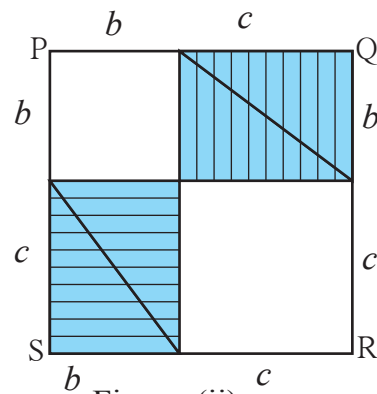


Figure (ii)

Observe the figures. In figure (i) we can see a square of side a units in the uncovered portion of square ABCD. In figure (ii) we see a square of side b and another of side c in the uncovered portion of the square PQRS.

In figure (i), area of square ABCD = $a^2 + 4 \times \text{area of right-angled triangle}$
 $= a^2 + 4 \times \frac{1}{2} bc$
 $= a^2 + 2bc$

$$\begin{aligned}
 \text{In figure (ii), area of square PQRS} &= b^2 + c^2 + 4 \times \text{area of right-angled triangle} \\
 &= b^2 + c^2 + 4 \times \frac{1}{2} bc \\
 &= b^2 + c^2 + 2bc
 \end{aligned}$$

Area of square ABCD = Area of square PQRS

$$\therefore a^2 + 2bc = b^2 + c^2 + 2bc$$

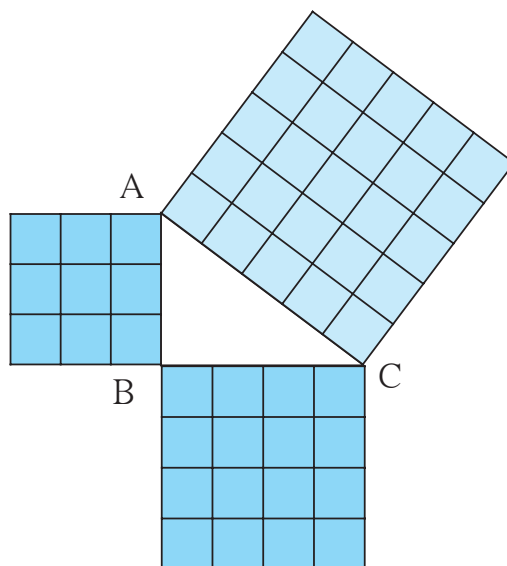
$$\therefore a^2 = b^2 + c^2$$



Let's discuss.

- Without using a protractor, can you verify that every angle of the vacant quadrilateral in figure (i) is a right angle ?

Activity : On a sheet of card paper, draw a right-angled triangle of sides 3 cm, 4 cm and 5 cm. Construct a square on each of the sides. Find the area of each of the squares and verify Pythagoras' theorem.



Given two sides of a right-angled triangle, you can find the third side, using Pythagoras' theorem.

Example In $\triangle ABC$, $\angle C = 90^\circ$, $l(AC) = 5$ cm and $l(BC) = 12$ cm. What is the length of seg (AB)?

Solution : In the right-angled triangle ABC, $\angle C = 90^\circ$.

Hence, side AB is the hypotenuse.

According to Pythagoras' theorem,

$$l(AB)^2 = l(AC)^2 + l(BC)^2$$

$$= 5^2 + 12^2$$

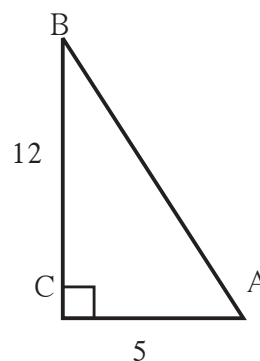
$$= 25 + 144$$

$$l(AB)^2 = 169$$

$$l(AB)^2 = 13^2$$

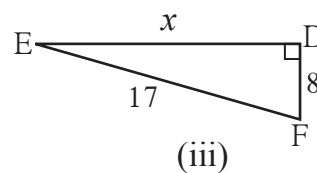
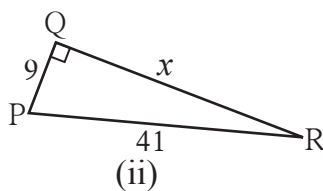
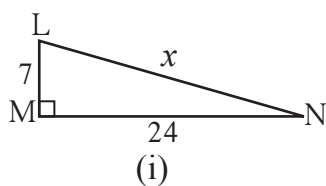
$$l(AB) = 13$$

\therefore Length of seg AB = 13 cm.



Practice Set 48

1. In the figures below, find the value of 'x'.



2. In the right-angled $\triangle PQR$, $\angle P = 90^\circ$. If $l(PQ) = 24$ cm and $l(PR) = 10$ cm, find the length of seg QR.
3. In the right-angled $\triangle LMN$, $\angle M = 90^\circ$. If $l(LM) = 12$ cm and $l(LN) = 20$ cm, find the length of seg MN.
4. The top of a ladder of length 15 m reaches a window 9 m above the ground. What is the distance between the base of the wall and that of the ladder ?



Let's learn.

If, in a triplet of natural numbers, the square of the biggest number is equal to the sum of the squares of the other two numbers, then the three numbers form a **Pythagorean triplet**. If the lengths of the sides of a triangle form such a triplet, then the triangle is a right-angled triangle.

Example Do the following numbers form a Pythagorean triplet : (7, 24, 25) ?

Solution: $7^2 = 49$, $24^2 = 576$, $25^2 = 625$

$$\therefore 49 + 576 = 625$$

$$\therefore 7^2 + 24^2 = 25^2$$

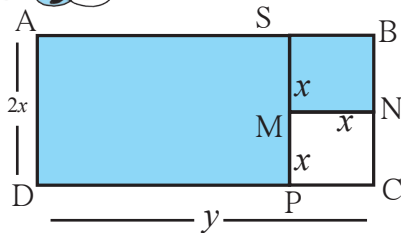
7, 24 and 25 is a Pythagorean triplet.

Activity: From the numbers 1 to 50, pick out Pythagorean triplets.

Practice Set 49

1. Find the Pythagorean triplets from among the following sets of numbers.
- | | |
|---------------|--------------|
| (i) 3, 4, 5 | (ii) 2, 4, 5 |
| (iii) 4, 5, 6 | (iv) 2, 6, 7 |
| (v) 9, 40, 41 | (vi) 4, 7, 8 |
2. The sides of some triangles are given below. Find out which ones are right-angled triangles?
- | | | | |
|----------------|-----------------|------------------|--------------------|
| (i) 8, 15, 17 | (ii) 11, 12, 15 | (iii) 11, 60, 61 | (iv) 1.5, 1.6, 1.7 |
| (v) 40, 20, 30 | | | |



**Let's recall.**

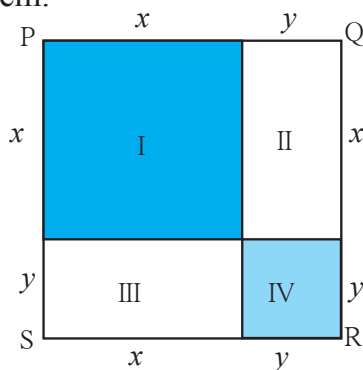
A rectangle ABCD is shown in the figure alongside. Its length is y units and its breadth, $2x$ units. A square of side x units is cut out from this rectangle. We can use operations on algebraic expressions to find the area of the shaded part. Let us write the area of rectangle ABCD as $A(\square ABCD)$

$$\begin{aligned}\text{Area of the shaded part} &= A(\square ABCD) - A(\square MNCP) \\ &= 2xy - x^2\end{aligned}$$

$$\begin{aligned}\text{Area of the shaded part} &= A(\square ASPD) + A(\square SBNM) \\ &= (y - x) \times 2x + x^2 \\ &= 2xy - 2x^2 + x^2 \\ &= 2xy - x^2\end{aligned}$$

**Let's learn.****The Expanded Form of the Square of a Binomial**

The product of algebraic expressions is called their 'expansion' or their 'expanded form'. There are some formulae which help in writing certain expansions. Let's consider some of them.

Activity I

- In the figure alongside, the side of the square PQRS is $(x + y)$.

$$\therefore A(\square PQRS) = (x + y)^2$$

The square PQRS is divided into 4 rectangles : I, II, III, IV

$A(\square PQRS) = \text{Sum of areas of rectangles I, II, III, IV.}$

$$\therefore A(\square PQRS) = A(\text{Rectangle I}) + A(\text{Rectangle II}) + A(\text{Rectangle III}) + A(\text{Rectangle IV})$$

$$(x + y)^2 = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$$

$$\therefore (x + y)^2 = x^2 + 2xy + y^2$$

Now, let us multiply $(x + y)^2$ as algebraic expressions.

$$(x + y)(x + y) = x(x + y) + y(x + y)$$

$$= x^2 + xy + yx + y^2 \quad \therefore (x + y)^2 = x^2 + 2xy + y^2$$

The expression obtained by squaring the binomial $(x + y)$ is equal to the expression obtained by finding the area of the square. Therefore, $(x + y)^2 = x^2 + 2xy + y^2$ is the formula for the expansion of the square of a binomial.

Activity II In the figure alongside, the square with side a is divided into 4 rectangles, namely, square with side $(a-b)$, square with side b and two rectangles of sides $(a-b)$ and b .

$$A(\text{square I}) + A(\text{rectangle II}) + A(\text{rectangle III}) + A(\text{square IV}) = A(\square PQRS)$$

$$(a-b)^2 + (a-b)b + (a-b)b + b^2 = a^2$$

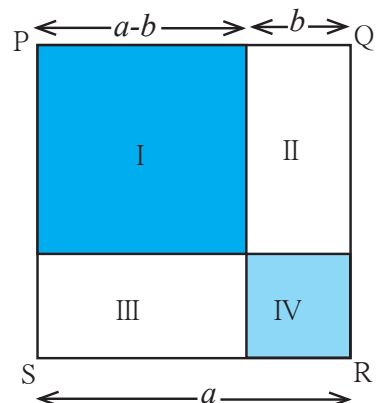
$$(a-b)^2 + 2ab - b^2 + b^2 = a^2$$

$$(a-b)^2 + 2ab - b^2 = a^2$$

$$\therefore (a-b)^2 = a^2 - 2ab + b^2$$

Let us multiply the algebraic expressions and obtain the formula.

$$\begin{aligned}(a-b)^2 &= (a-b) \times (a-b) \\ &= a(a-b) - b(a-b) \\ &= a^2 - ab - ab + b^2 \\ &= a^2 - 2ab + b^2\end{aligned}$$



Now I know!

$$\bullet (a+b)^2 = a^2 + 2ab + b^2$$

$$\bullet (a-b)^2 = a^2 - 2ab + b^2$$

We can verify the formulae by substituting a and b with any numbers.

Thus, if $a = 5$, $b = 3$, then

$$(a+b)^2 = (5+3)^2 = 8^2 = 64$$

$$\begin{aligned}a^2 + 2ab + b^2 &= 5^2 + 2 \times 5 \times 3 + 3^2 \\ &= 25 + 30 + 9 = 64\end{aligned}$$

$$(a-b)^2 = (5-3)^2 = 2^2 = 4$$

$$\begin{aligned}a^2 - 2ab + b^2 &= 5^2 - 2 \times 5 \times 3 + 3^2 \\ &= 25 - 30 + 9 = 4\end{aligned}$$

Use the given values to verify the formulae for squares of binomials.

(i) $a = -7$, $b = 8$

(ii) $a = 11$, $b = 3$

(iii) $a = 2.5$, $b = 1.2$

Expand.

Example $(2x + 3y)^2$

$$\begin{aligned}&= (2x)^2 + 2(2x) \times (3y) + (3y)^2 \\ &= 4x^2 + 12xy + 9y^2\end{aligned}$$

Example $(5x - 4)^2$

$$\begin{aligned}&= (5x)^2 - 2(5x) \times (4) + 4^2 \\ &= 25x^2 - 40x + 16\end{aligned}$$

Example $(51)^2$

$$\begin{aligned}&= (50 + 1)^2 \\ &= 50^2 + 2 \times 50 \times 1 + 1 \times 1 \\ &= 2500 + 100 + 1 \\ &= 2601\end{aligned}$$

Example $(98)^2$

$$\begin{aligned}&= (100 - 2)^2 \\ &= 100^2 - 2 \times 100 \times 2 + 2^2 \\ &= 10000 - 400 + 4 \\ &= 9604\end{aligned}$$

Practice Set 50

1. Expand.

$$\begin{array}{llll} \text{(i)} (5a + 6b)^2 & \text{(ii)} \left(\frac{a}{2} + \frac{b}{3}\right)^2 & \text{(iii)} (2p - 3q)^2 & \text{(iv)} \left(x - \frac{2}{x}\right)^2 \\ \text{(v)} (ax + by)^2 & \text{(vi)} (7m - 4)^2 & \text{(vii)} \left(x + \frac{1}{2}\right)^2 & \text{(viii)} \left(a - \frac{1}{a}\right)^2 \end{array}$$

2. Which of the options given below is the square of the binomial $\left(8 - \frac{1}{x}\right)$?

$$\begin{array}{llll} \text{(i)} 64 - \frac{1}{x^2} & \text{(ii)} 64 + \frac{1}{x^2} & \text{(iii)} 64 - \frac{16}{x} + \frac{1}{x^2} & \text{(iv)} 64 + \frac{16}{x} + \frac{1}{x^2} \end{array}$$

3. Of which of the binomials given below is $m^2n^2 + 14mnpq + 49p^2q^2$ the expansion?

$$\begin{array}{llll} \text{(i)} (m + n)(p + q) & \text{(ii)} (mn - pq) & \text{(iii)} (7mn + pq) & \text{(iv)} (mn + 7pq) \end{array}$$

4. Use an expansion formula to find the values.

$$\begin{array}{llll} \text{(i)} (997)^2 & \text{(ii)} (102)^2 & \text{(iii)} (97)^2 & \text{(iv)} (1005)^2 \end{array}$$



Let's learn.

* **Expansion of $(a + b)(a - b)$**

$$\begin{aligned} (a + b)(a - b) &= (a + b) \times (a - b) \\ &= a(a - b) + b(a - b) \\ &= a^2 - ab + ba - b^2 \\ &= a^2 - b^2 \\ (a + b)(a - b) &= a^2 - b^2 \end{aligned}$$



Now I know!

$$(a + b)(a - b) = a^2 - b^2$$

Example $(3x + 4y)(3x - 4y) = (3x)^2 - (4y)^2 = 9x^2 - 16y^2$

Example $102 \times 98 = (100 + 2)(100 - 2) = (100)^2 - (2)^2 = 10000 - 4 = 9996$

Practice Set 51

1. Use the formula to multiply the following.

$$\begin{array}{ll} \text{(i)} (x + y)(x - y) & \text{(ii)} (3x - 5)(3x + 5) \\ \text{(iii)} (a + 6)(a - 6) & \text{(iv)} \left(\frac{x}{5} + 6\right)\left(\frac{x}{5} - 6\right) \end{array}$$

2. Use the formula to find the values.

$$\begin{array}{llll} \text{(i)} 502 \times 498 & \text{(ii)} 97 \times 103 & \text{(iii)} 54 \times 46 & \text{(iv)} 98 \times 102 \end{array}$$

**Let's learn.****Factorising Algebraic Expressions**

We have learnt to factorise whole numbers. Now let us learn to factorise algebraic expressions. First, let us factorise a monomial.

$15 = 3 \times 5$, that is, 3 and 5 are factors of 15.

Similarly, $3x = 3 \times x$, Hence, 3 and x are factors of $3x$

Consider $5t^2$. $5t^2 = 5 \times t^2 = 5 \times t \times t$

1, 5, t , t^2 , $5t$, $5t^2$ are all factors of $5t^2$.

$6ab^2 = 2 \times 3 \times a \times b \times b$

When factorising a monomial, first factorise the coefficient if possible and then factorise the part with variables.

Practice Set 52

⊙ Factorise the following expressions and write them in the product form.

(i) $201 a^3 b^2$, (ii) $91 xyt^2$, (iii) $24 a^2 b^2$, (iv) tr^2s^3

**Let's learn.****Factorising a Binomial**

4, x and y are factors of every term in the binomial $4xy + 8xy^2$

$\therefore 4xy + 8xy^2 = 4(xy + 2xy^2) = 4x(y + 2xy) = 4xy(1 + 2y)$

We can factorise a binomial by identifying the factors common to both terms and writing them outside the brackets in product form.

This is how we factorise $9a^2bc + 12abc^2 = 3(3a^2bc + 4abc^2) = 3abc(3a + 4c)$

$(a + b)(a - b) = a^2 - b^2$ is a formula we have already learnt.

Hence, we also get the factors $a^2 - b^2 = (a + b)(a - b)$

Factorise:

Example $a^2 - 4b^2 = a^2 - (2b)^2$
 $= (a + 2b)(a - 2b)$

Example $3a^2 - 27b^2 = 3(a^2 - 9b^2)$
 $= 3(a + 3b)(a - 3b)$

Practice Set 53

⊙ Factorise the following expressions.

(i) $p^2 - q^2$

(ii) $4x^2 - 25y^2$

(iii) $y^2 - 4$

(iv) $p^2 - \frac{1}{25}$

(v) $9x^2 - \frac{1}{16}y^2$

(vi) $x^2 - \frac{1}{x^2}$

(vii) $a^2b - ab$

(viii) $4x^2y - 6x^2$

(ix) $\frac{1}{2}y^2 - 8z^2$

(x) $2x^2 - 8y^2$





Let's learn.

Average

The following table shows how many minutes Asmita took to cycle to school every morning, from Monday to Saturday.



Day	Mon	Tue	Wed	Thu	Fri	Sat
Minutes	20	20	22	18	18	20

We see from the table that she takes 18 minute on some days, 20 on others and even 22 minutes on one day. If we consider these six school days, what would you say is the approximate time she takes to cycle to school?

In mathematics, to make such an estimate, we find the 'average'. If we add together the number of minutes required on each day and divide the sum by six, the number we get is, approximately, the time required every day. It is the 'average' of all six numbers.

$$\begin{aligned}
 \text{Average} &= \frac{\text{Sum of the number of minutes taken to cycle to school on each of six days}}{\text{Total days}} \\
 &= \frac{20 + 20 + 22 + 18 + 18 + 20}{6} = \frac{118}{6} = 19 \frac{2}{3}
 \end{aligned}$$

On an average, Asmita takes $19 \frac{2}{3}$ minutes to cycle to school every day.

Example A school conducted a survey to find out how far their students live from the school. Given below is the distance of the houses of six of the students from the school. Let us find their average distance from the school.

950 m, 800 m, 700 m, 1.5 km, 1 km, 750 m

Solution : To find the average, we must first express all the distances in the same units.

$$\text{Average} = \frac{\text{Sum of the distance between home and school for six students}}{\text{Total number of students}}$$

$$\begin{aligned}
 1 \text{ km} &= 1000 \text{ m} \\
 1.5 \text{ km} &= 1500 \text{ m}
 \end{aligned}$$

$$= \frac{950 + 800 + 700 + 1500 + 1000 + 750}{6} = \frac{5700}{6} = 950 \text{ m}$$

The average distance at which the students live from the school is 950 m.



Let's discuss.

Example Rutuja practised skipping with a rope all seven days of a week. The number of times she jumped the rope in one minute every day is given below.

60, 62, 61, 60, 59, 63, 58

$$\text{Average} = \frac{\text{Sum of the number of jumps on seven days}}{\text{Total number of days}}$$

$$= \frac{\square + \square + \square + \square + \square + \square + \square}{7} = \frac{\square}{\square}$$

Average number of jumps per minute = 60.42



The samples that we have of the quantity we are measuring are called 'readings' or 'scores'.

We know that the number of jumps will be counted in natural numbers. Never will there be a fractional number of jumps. However, their average can be a fractional number.



Now I know!

$$\text{Average} = \frac{\text{Sum of all scores in the given data}}{\text{Total number of scores}}$$

- Activity :** ★ Make groups of 10 children, in your class. Find the average height of the children in each group.
- ★ With the help of your class teacher, note the daily attendance for a week and find the average attendance.

Practice Set 54

- The daily rainfall for each day of a week in a certain city is given in millimetres. Find the average rainfall during the week.
9, 11, 8, 20, 10, 16, 12
- During the annual function of a school, a Women's Self-help Group had set up a snacks stall. Their sales every hour were worth ₹ 960, ₹ 830, ₹ 945, ₹ 800, ₹ 847, ₹ 970 respectively. What was the average of the hourly sales?
- The annual rainfall in Vidarbha in five years is given below. What is the average rainfall for those 5 years ?
900 mm, 650 mm, 450 mm, 733 mm, 400 mm
- A farmer bought some sacks of animal feed. The weights of the sacks are given below in kilograms. What is the average weight of the sacks ?
49.8, 49.7, 49.5, 49.3, 50, 48.9, 49.2, 48.8



Let's learn.

Frequency Distribution Table

Sometimes, in collected data, some scores appear again and again. **The number of times a particular score occurs in a data is called the frequency of that score.** In such cases a frequency table is made with three columns, one each for the score, the tally marks and the frequency.

1. In the first column, scores are entered in ascending order. For example, enter 1, 2, 3, 4, 5, 6 in order one below the other.
2. Read the scores in the data in serial order and enter a tally mark 'I' for each in the second column of the table in front of that score, e.g. if you read the score '3', make a tally mark in front of 3 in the second column. Place four tally marks like this IIII , but make the fifth one like this IIII . It makes it easier to count the total number of tally marks.
3. Count the total number of tally marks in front of each score and enter the number in the next, i.e. third, column. This number is the frequency of the score.
4. Lastly, add all the frequencies. Their sum is denoted by the letter N. This sum is equal to the total number of scores.

Making a Frequency Table of the Given Information/ Data

Example The distance at which some children live from their school is given below in kilometres.

1, 3, 2, 4, 5, 4, 1, 3, 4, 5, 6, 4, 6, 4, 6

Let us see how to make a frequency table of this data.

Scores	Tally marks	Frequency
1	II	2
2	I	1
3	II	2
4	IIII	5
5	II	2
6	III	3
Total frequency		$N = 15$

We strike off a score to indicate that it has been counted. The list of scores below shows that the first three scores have already been counted.

(~~1~~, ~~3~~, ~~2~~, 4, 5, 4, 1, 3, 4, 5, 6, 4, 6, 4, 6)

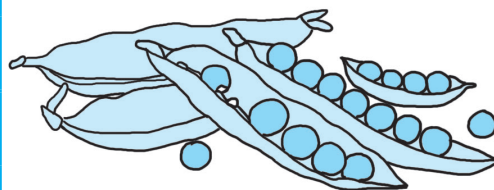


My friend, Maths : At home, in the market.

Priya's mother bought some peas and began to shell them. Priya was sitting nearby studying her maths lesson and she observed that some of the peapods had just 4 peas while some had 7. So, she took 50 of the pods and, as she shelled them, she noted down the number of peas in each of the pods.

She also made a frequency table of the peas in the pods.

Number of peas in a pod	Tally marks	Frequency
2		8
3		15
4		12
5		2
6		7
7		3
8		3
Total frequency		N = 50



4, 3, 2, 4, 3, 4, 3, 3, 2, 8
2, 3, 3, 4, 3, 4, 4, 5, 2, 8
8, 2, 5, 3, 4, 4, 3, 6, 2, 3
4, 4, 3, 3, 2, 6, 4, 4, 7, 2
3, 6, 3, 6, 6, 6, 7, 6, 7, 3

Mother : Can you find out the average number of peas in a pod?

Priya : I will have to add 50 numbers and then divide their sum by 50. It will be tedious work.

Mother : Let's make it easier. You can tell from the frequency table how many pods had 2 peas, how many had 3 and so on, right?

Priya : Yes! 8 pods had 2 peas each, 15 had 3, 12 had 4.... Oh, now I see. If I multiply and find the products like 2×8 , 3×15 , 4×12 and then add all the products I will get the sum of all those 50 numbers.

Mother : It is easier to do seven simple multiplications and add them up, isn't it? This is how the frequency table proves useful when we have a huge amount of data.

Priya : The sum of all scores was 206.
So, their average = $\frac{206}{50} = 4.12$.

Mother : Peas in a pod are always found in whole numbers, but the average can be a fraction. In this case, we can say that there were about 4 peas in every pod.



Now I know!

- A simple way to tabulate scores is by using tally marks.
- A table in which the number of tally marks indicates the frequency is called a frequency table.
- When the number of scores is very large, a frequency table is used to find their average.

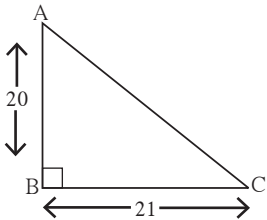
Practice Set 55

1. The height of 30 children in a class is given in centimetres. Draw up a frequency table of this data.
131, 135, 140, 138, 132, 133, 135, 133, 134, 135, 132, 133, 140, 139, 132, 131, 134, 133, 140, 140, 139, 136, 137, 136, 139, 137, 133, 134, 131, 140
2. In a certain colony, there are 50 families. The number of people in every family is given below. Draw up the frequency table.
5, 4, 5, 4, 5, 3, 3, 3, 4, 3, 4, 2, 3, 4, 2, 2, 2, 2, 4, 5, 1, 3, 2, 4, 5, 3, 3, 2, 4, 4, 2, 3, 4, 3, 4, 2, 3, 4, 5, 3, 2, 3, 2, 3, 4, 5, 3, 2, 3, 2
3. A dice was cast 40 times and each score noted is given below. Draw up a frequency table for this data.
3, 2, 5, 6, 4, 2, 3, 1, 6, 6, 2, 3, 5, 3, 5, 3, 4, 2, 4, 5, 4, 2, 6, 3, 3, 2
4, 3, 3, 4, 1, 4, 3, 3, 2, 2, 5, 3, 3, 4
4. The number of chapatis that 30 children in a hostel need at every meal is given below. Make a frequency table for these scores.
3, 2, 2, 3, 4, 5, 4, 3, 4, 5, 2, 3, 4, 3, 2, 5, 4, 4, 4, 3, 3, 2, 2, 2, 3, 4, 3, 2, 3, 2

The 'average' is a useful figure in the study of all branches of science including medicine, geography, economics, social science, etc.



Miscellaneous Problems : Set 2

- Angela deposited 15000 rupees in a bank at a rate of 9 p.c.p.a. She got simple interest amounting to 5400 rupees. For how many years had she deposited the amount?
- Ten men take 4 days to complete the task of tarring a road. How many days would 8 men take?
- Nasruddin and Mahesh invested ₹ 40,000 and ₹ 60,000 respectively to start a business. They made a profit of 30%. How much profit did each of them make?
- The diameter of a circle is 5.6 cm. Find its circumference.
- Expand.
(i) $(2a - 3b)^2$ (ii) $(10 + y)^2$ (iii) $\left(\frac{p}{3} + \frac{q}{4}\right)^2$ (iv) $\left(y - \frac{3}{y}\right)^2$
- Use a formula to multiply.
(i) $(x - 5)(x + 5)$ (ii) $(2a - 13)(2a + 13)$
(iii) $(4z - 5y)(4z + 5y)$ (iv) $(2t - 5)(2t + 5)$
- The diameter of the wheel of a cart is 1.05 m. How much distance will the cart cover in 1000 rotations of the wheel?
- The area of a rectangular garden of length 40 m, is 1000 sqm. Find the breadth of the garden and its perimeter. The garden is to be enclosed by 3 rounds of fencing, leaving an entrance of 4 m. Find the cost of fencing the garden at a rate of 250 rupees per metre.
- 

From the given figure, find the length of hypotenuse AC and the perimeter of $\triangle ABC$.
- If the edge of a cube is 8 cm long, find its total surface area.
- Factorise. $365y^4z^3 - 146y^2z^4$

Multiple Choice Questions

Choose the right answers from the options given for each of the following questions.

- If the average of the numbers 33, 34, 35, x , 37, 38, 39 is 36, what is the value of x ?
(i) 40 (ii) 32 (iii) 42 (iv) 36
- The difference of the squares, $(61^2 - 51^2)$ is equal to
(i) 1120 (ii) 1230 (iii) 1240 (iv) 1250
- If 2600 rupees are divided between Sameer and Smita in the proportion 8 : 5, the share of each is and respectively.
(i) ₹ 1500, ₹ 1100 (ii) ₹ 1300, ₹ 900
(iii) ₹ 800, ₹ 500 (iv) ₹ 1600, ₹ 1000

ANSWERS

Practice Set 1 1. --- 2. --- 3. In the interior of the triangle 4. On the hypotenuse of right-angled triangle 5. To draw circumcentre of the triangle.

Practice Set 2 --- **Practice Set 3** ---

Practice Set 4 --- **Practice Set 5** ---

Practice Set 6 : 1. (i) $\text{Seg MG} \cong \text{Seg GR}$

(ii) $\text{Seg MG} \cong \text{Seg NG}$

(iii) $\text{Seg GC} \cong \text{Seg GB}$

(iv) $\text{Seg GE} \cong \text{Seg GR}$

2. (i) $\text{Seg AB} \cong \text{Seg WA}$

(ii) $\text{Seg AP} \cong \text{Seg YC}$

(iii) $\text{Seg AC} \cong \text{Seg PY}$

(iv) $\text{Seg PW} \cong \text{Seg BY}$

(v) $\text{Seg YA} \cong \text{Seg YQ}$

(vi) $\text{Seg BW} \cong \text{Seg ZX}$

(There may be many correct answers for each of the above questions.)

Practice Set 7 : $\odot \angle AOB \cong \angle BOC$

$\angle AOB \cong \angle RST \quad \angle AOC \cong \angle PQR$

$\angle DOC \cong \angle LMN \quad \angle BOC \cong \angle RST$

Practice Set 8 : \odot (i) 35 (ii) -54 (iii) -36

(iv) -56 (v) 124 (vi) 84 (vii) 441

(viii) -105

Practice Set 9 : 1. (i) -6 (ii) $\frac{-7}{2}$ (iii) $\frac{-3}{4}$

(iv) $\frac{-2}{3}$ (v) $\frac{-17}{4}$ (vi) 6 (vii) $\frac{5}{3}$ (viii) $\frac{-1}{6}$

(ix) $\frac{6}{5}$ (x) $\frac{1}{63}$ 2. $24 \div 5$, $72 \div 15$,

$-48 \div (-10)$ etc. 3. $-5 \div 7$, $-15 \div 21$, $20 \div (-28)$ etc.

Practice Set 10 : 1. 1 2. 4, 5 and 17, 19

3. 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 Total prime numbers 16

4. 59 and 61, 71 and 73 5. (2,3), (5,7),

(11,12), (17,19), (29,30) etc. 6. 2

Practice Set 11 : \odot (i) $2 \times 2 \times 2 \times 2 \times 2$

(ii) 3×19 (iii) 23 (iv) $2 \times 3 \times 5 \times 5$

(v) $2 \times 2 \times 2 \times 3 \times 3 \times 3$

(vi) $2 \times 2 \times 2 \times 2 \times 13$ (vii) $3 \times 3 \times 5 \times 17$

(viii) $2 \times 3 \times 3 \times 19$ (ix) 13×29 (x) 13×43

Practice Set 12 : 1. (i) 5 (ii) 8 (iii) 5

(iv) 1 (v) 2 (vi) 7 (vii) 3 (viii) 3

(ix) 1 (x) 21

2. (i) HCF 25, Simplest form $\frac{11}{21}$

(ii) HCF 19, Simplest form $\frac{4}{7}$

(iii) HCF 23, Simplest form $\frac{7}{3}$

Practice Set 13 : 1. (i) 60 (ii) 120 (iii) 288

(iv) 60 (v) 3870 (vi) 90 (vii) 1365 (viii) 180

(ix) 567 (x) 108

2. (i) 1; 1184 (ii) 1; 2346 (iii) 15; 60

(iv) 9; 126 (v) 26; 312

Practice Set 14 : 1. (i) 30 (ii) 40, 20

2. (i) 14; 28 (ii) 16; 32 (iii) 17; 510

(iv) 23; 69 (v) 7; 588

3. (i) 252 (ii) 150 (iii) 1008 (iv) 60 (v) 240

4. 365 5. (i) $\frac{12}{11}$ (ii) $\frac{17}{19}$ (iii) $\frac{23}{29}$ 6. 144

7. 255 8. 14m 9. 18 and 20

Practice Set 15 : 1. Points in the interior : R, C, N, X

Points in the exterior : T, U, Q, V, Y

Points on the arms of the angles : A, W, G, B

2. $\angle ANB$ and $\angle BNC$, $\angle BNC$ and $\angle ANC$, $\angle ANC$ and $\angle ANB$, $\angle PQR$ and $\angle PQT$

3. (i) The pairs are adjacent. (ii) and (iii) are not adjacent because the interiors are not separate. (iv) The pairs are adjacent.

Practice Set 16 : 1. (i) 50° (ii) 27° (iii) 45°

(iv) 35° (v) 70° (vi) 0° (vii) $(90-x)^\circ$

2. 20° and 70°

Practice Set 17 : 1. (i) 165° (ii) 95° (iii) 60°
(iv) 143° (v) 72° (vi) 180° (vii) $(180-a)^\circ$

2. Pairs of complementary angles : (i) $\angle B$ and $\angle N$ (ii) $\angle D$ and $\angle F$ (iii) $\angle Y$ and $\angle E$

Pairs of supplementary angles : (i) $\angle B$ and $\angle G$ (ii) $\angle N$ and $\angle J$ 3. $\angle X$ and $\angle Z$ are complementary angles. 4. 65° and 25° 5. (i) $\angle P$ and $\angle M$ (ii) $\angle T$ and $\angle N$ (iii) $\angle P$ and $\angle T$ (iv) $\angle M$ and $\angle N$ (v) $\angle P$ and $\angle N$ (vi) $\angle M$ and $\angle T$ 6. 160°

7. $m\angle A = (160-x)^\circ$

Practice Set 18 : 1. Ray PL and Ray PM;

Ray PN and Ray PT. 2. No. Because the rays do not form a straight line.

Practice Set 19 : ---

Practice Set 20 : 1. $m\angle APB = 133^\circ$,

$m\angle BPC = 47^\circ$, $m\angle CPD = 133^\circ$,

2. $m\angle PMS = (180-x)^\circ$, $m\angle SMQ = x^\circ$,

$m\angle QMR = (180-x)^\circ$,

Practice Set 21 : 1. $m\angle A = m\angle B = 70^\circ$

2. 40° , 60° , 80° 3. $m\angle ACB = 34^\circ$,

$m\angle ACD = 146^\circ$, $m\angle A = m\angle B = 73^\circ$

Practice Set 22 : 1. (i) $\frac{71}{252}$ (ii) $\frac{67}{15}$

(iii) $\frac{430}{323}$ (iv) $\frac{255}{77}$ 2. (i) $\frac{16}{77}$ (ii) $\frac{14}{45}$ (iii) $-\frac{13}{6}$

(iv) $\frac{7}{6}$ 3. (i) $\frac{6}{55}$ (ii) $\frac{16}{25}$ (iii) $-\frac{2}{3}$ (iv) 0

4. (i) $\frac{5}{2}$ (ii) $-\frac{8}{3}$ (iii) $-\frac{39}{17}$ (iv) $\frac{1}{7}$ (v) $-\frac{3}{22}$

5. (i) $\frac{4}{3}$ (ii) $\frac{100}{121}$ (iii) $\frac{7}{4}$ (iv) $-\frac{1}{6}$ (v) $\frac{2}{5}$

(vi) $-\frac{10}{7}$ (vii) $-\frac{9}{88}$ (viii) $\frac{25}{2}$

Practice Set 23 : \odot (i) $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ (ii) $\frac{23}{30}, \frac{22}{30}, \frac{21}{30}$

(iii) $-\frac{9}{15}, -\frac{7}{15}, \frac{4}{15}$ (iv) $\frac{6}{9}, 0, -\frac{4}{9}$ (v) $-\frac{2}{4}, -\frac{1}{4}, \frac{3}{4}$

(vi) $\frac{17}{24}, \frac{11}{24}, -\frac{13}{24}$ (vii) $\frac{6}{7}, \frac{8}{7}, \frac{9}{7}$

(viii) $-\frac{1}{8}, -\frac{2}{8}, -\frac{5}{8}$ etc.

Practice Set 24 : \odot 1. 3.25 2. -0.875 3. 7.6

4. $0.41\dot{6}$ 5. $3.\overline{142857}$ 6. $1.\dot{3}$ 7. $0.\dot{7}$

Practice Set 25 : 1. 149 2. 0 3. 4 4. 60

5. $\frac{17}{20}$

Practice Set 26 : 1. -- 2. (i) 1024 (ii) 125

(iii) 2401 (iv) -216 (v) 729 (vi) 8 (vii) $\frac{64}{125}$

(viii) $\frac{1}{16}$

Practice Set 27 : \odot (i) 7^6 (ii) $(-11)^7$ (iii) $\left(\frac{6}{7}\right)^8$

(iv) $\left(-\frac{3}{2}\right)^8$ (v) $(a)^{23}$ (vi) $\left(\frac{p}{5}\right)^{10}$

Practice Set 28 : 1. (i) a^2 (ii) m^{-3} (iii) p^{-10}

(iv) 1 2. (i) 1 (ii) 49 (iii) $\frac{4}{5}$ (iv) 16

Practice Set 29 : 1. (i) $\left(\frac{15}{12}\right)^{12}$ (ii) 3^{-8}

(iii) $\left(\frac{1}{7}\right)^{-12}$ (iv) $\left(\frac{2}{5}\right)^6$ (v) 6^{20} (vi) $\left(\frac{6}{7}\right)^{10}$

(vii) $\left(\frac{2}{3}\right)^{-20}$ (viii) $\left(\frac{5}{8}\right)^{-6}$ (ix) $\left(\frac{3}{4}\right)^6$ (x) $\left(\frac{2}{5}\right)^{-6}$

2. (i) $\left(\frac{7}{2}\right)^2$ (ii) $\left(\frac{3}{11}\right)^5$ (iii) $\left(\frac{6}{1}\right)^3$ or 6^3

(iv) $\frac{1}{y^4}$

Practice Set 30 : 1. (i) 25 (ii) 35 (iii) 17

(iv) 64 (v) 33

Practice Set 31 : ---

Practice Set 32 : \odot Monomials = $7x$; a ; 4

Binomials = $5y - 7z$; $5m - 3$

Trinomials = $3x^3 - 5x^2 - 11$; $3y^2 - 7y + 5$

Polynomials = $1 - 8a - 7a^2 - 7a^3$

Practice Set 33 : \odot (i) $22p + 18q$

- (ii) $18a + 24b + 21c$ (iii) $19x^2 - 20y^2$
 (iv) $-11a^2b^2 + 44c$ (v) $3y^2 - 8y + 9$
 (vi) $4y^2 + 10y - 8$

Practice Set 34 : (i) $xy + 7z$
 (ii) $4x + 2y + 4z$ (iii) $-12x^2 + 16xy + 20y^2$
 (iv) $-10x^2 + 24xy + 16y^2$
 (v) $-12x + 30z - 19y$

Practice Set 35 : 1. (i) $288x^2y^2$ (ii) $92xy^3z^2$
 (iii) $48ac + 68bc$ (iv) $36x^2 + 73xy + 35y^2$
 2. $(40x^2 + 49x + 15)$ sqcm

Practice Set 36 : 1. $-2(7x + 12y)$

2. $-345x^5y^4z^3$ 3. (i) 1 (ii) $\frac{5}{2}$ (iii) 1 (iv) 3
 (v) -5 (vi) $\frac{69}{5}$ 4. 16 years, 11 years 5. 130
 6. 30 Notes 7. 132, 66

Miscellaneous Problems : Set 1 : 1. (i) 80
 (ii) -6 (iii) -48 (iv) 25 (v) 8 (vi) -100
 2. (i) 15; 675 (ii) 38; 228 (iii) 17; 1683
 (iv) 8; 96 3. (i) $\frac{14}{17}$ (ii) $\frac{13}{11}$ (iii) $\frac{3}{4}$
 4. (i) 28 (ii) 15 (iii) 36 (iv) 45
 (v) 16 5. -- 6. (i) 77 (ii) 25 (iii) $\frac{49}{24}$
 (iv) 1026 7. (i) $\frac{41}{48}$ (ii) $\frac{23}{20}$ (iii) -8
 (iv) $\frac{63}{20}$ 8. -- 9. -- 10. -- 11. -- 12. --
 13. (i) 55° (ii) $(90 - a)^\circ$ (iii) 68°
 (iv) $(50 + x)^\circ$ 14. (i) 69° (ii) 133° (iii) 0°
 (iv) $(90 + x)^\circ$ 15. -- 16. (i) 110° (ii) 55°
 (iii) 55° 17. (i) 5^7 (ii) $\left(\frac{3}{2}\right)^3$ (iii) $\left(\frac{7}{2}\right)^2$
 (iv) $\left(\frac{4}{5}\right)^3$ 18. (i) 1 (ii) $\frac{1}{1000}$ (iii) 64
 (iv) 16 19. (i) $8a + 10b - 13c$
 (ii) $21x^2 - 10xy - 16y^2$ (iii) $18m - n$

- (iv) $2m - 19n + 11p$ 20. (i) $x = -10$
 (ii) $y = 5$

Multiple choice questions : 1. Incentre

2. $\left(\frac{7}{3}\right)^{12}$ 3. 3 4. $\frac{3}{2}$ 5. $10 \times 3 + (5 + 2)$

Practice Set 37 : 1. ₹ 240 2. 32 bunches of feed 3. 18 Kg 4. ₹ 24000 5. ₹ 104000

Practice Set 38 : 1. 10 days; 4 days
 2. 50 pages 3. 2 hours; 3 hours 4. 20 days

Practice Set 39 : 1. ₹ 12800; ₹ 16000
 2. ₹ 10000; ₹ 24000 3. ₹ 38000; ₹ 9120
 4. ₹ 147; ₹ 343 5. ₹ 54000; ₹ 15120

Practice Set 40 : 1. ₹ 1770
 2. ₹ 25000; ₹ 375000 3. ₹ 14875 4. ₹ 3600
 5. ₹ 180000

Practice Set 41 : 1. 10% 2. ₹ 300 3. 5 years
 4. ₹ 41000 5. (i) ₹ 882, ₹ 5082
 (ii) ₹ 5000, ₹ 6200 (iii) 2 years, ₹ 8800
 (iv) ₹ 12000, 10 years (v) ₹ 19200, ₹ 21600

Practice Set 42 : 1. (i) 14 cm; 44 cm
 (ii) 14 cm; 88 cm (iii) 98 cm; 196 cm
 (iv) 11.55 cm; 23.1 cm 2. 28 cm
 3. ₹ 56320 4. 250 rotations

Practice Set 43 : 1. 240°
 2. Names of minor arcs - arc PXQ, arc PR, arc RY, arc XP, arc XQ, arc QY
 Names of major arcs - arc PYQ, arc PQR, arc RQY, arc XQP, arc QRX
 Names of semicircular arcs - arc QPR, arc QYR 3. 250°

Practice Set 44 : 1. 2 times 2. 3 times
 3. 90 m 4. 8 m

Practice Set 45 : 1. 144 sqcm 2. 75 sqcm

3. 46 cm 4. 9 times

Practice Set 46 : 1. 1170 sqcm 2. 8.64 sqcm

3. ₹ 2302750 4. 800 tiles; 3200 tiles

5. 156 m; 845 sqm

Practice Set 47 : 1. (i) 54 sqcm (ii) 150 sqcm
(iii) 311.04 sqm (iv) 277.44 sqm

(v) 181.5 sqm 2. (i) 460 sqcm (ii) 58.8 sqcm

(iii) 31.6 sqm (iv) 171 sqcm 3. 39.5 sqcm

4. 6.5 sqm, ₹ 1950

Practice Set 48 : 1. (i) 25 units (ii) 40 units

(iii) 15 units 2. 26 cm 3. 16 cm 4. 12 m

Practice Set 49 : 1. (i) Yes. (ii) No. (iii) No.

(iv) No. (v) Yes. (vi) No.

2. (i) Yes. (ii) No. (iii) Yes. (iv) No. (v) No.

Practice Set 50 : 1. (i) $25a^2 + 60ab + 36b^2$

(ii) $\frac{a^2}{4} + \frac{ab}{3} + \frac{b^2}{9}$ (iii) $4p^2 - 12pq + 9q^2$

(iv) $x^2 - 4 + \frac{4}{x^2}$ (v) $a^2x^2 + 2abxy + b^2y^2$

(vi) $49m^2 - 56m + 16$ (vii) $x^2 + x + \frac{1}{4}$

(viii) $a^2 - 2 + \frac{1}{a^2}$ 2. $64 - \frac{16}{x} + \frac{1}{x^2}$

3. $(mn + 7pq)^2$ 4. (i) 994009 (ii) 10404

(iii) 9409 (iv) 1010025

Practice Set 51 : 1. (i) $x^2 - y^2$ (ii) $9x^2 - 25$

(iii) $a^2 - 36$ (iv) $\frac{x^2}{25} - 36$ 2. (i) 249996

(ii) 9991 (iii) 2484 (iv) 9996

Practice Set 52 : (i) $3 \times 67 \times a \times a \times a \times b \times b$

(ii) $13 \times 7 \times x \times y \times t \times t$

(iii) $2 \times 2 \times 2 \times 3 \times a \times a \times b \times b$

(iv) $t \times r \times r \times s \times s \times s$

Practice Set 53 : (i) $(p+q)(p-q)$

(ii) $(2x+5y)(2x-5y)$ (iii) $(y+2)(y-2)$

(iv) $\left(p + \frac{1}{5}\right)\left(p - \frac{1}{5}\right)$ (v) $\left(3x + \frac{1}{4}y\right)\left(3x - \frac{1}{4}y\right)$

(vi) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$ (vii) $ab(a-1)$

(viii) $2x^2(2xy-3x)$ (ix) $\frac{1}{2}(y+4z)(y-4z)$

(x) $2(x+2y)(x-2y)$

Practice Set 54 : 1. 12.29 mm 2. ₹ 892

3. 626.6 mm 4. 49.4 kg

Practice Set 55 : 1.

Height	131	132	133	134	135	136	137	138	139	140	Total
Children	3	3	5	3	3	2	2	1	3	5	30

2.

People	1	2	3	4	5	Total
Families	1	13	16	13	7	50

3.

Score	1	2	3	4	5	6	Total
Frequency	2	8	13	8	5	4	40

4.

Chapatis	2	3	4	5	Total
Children	9	10	8	3	30

Miscellaneous Problems : Set 2 : 1. 4 years

2. 5 days 3. ₹ 12000; ₹ 18000 4. 17.6 cm

5. (i) $4a^2 - 12ab + 9b^2$ (ii) $100 + 20y + y^2$

(iii) $\frac{p^2}{9} + \frac{pq}{6} + \frac{q^2}{16}$ (iv) $y^2 - 6 + \frac{9}{y^2}$

6. (i) $x^2 - 25$ (ii) $4a^2 - 169$ (iii) $16z^2 - 25y^2$
(iv) $4t^2 - 25$ 7. 3.3 km

8. 25 m; 130 m; ₹ 94500 9. 29 Units;

70 Units 10. 384 cm² 11. $73y^2z^3(5y^2 - 2z)$

Multiple choice questions : 1. 36 2. 1120

3. ₹ 1600, ₹ 1000.



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